Hurst exponents for short time series

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A concept called balanced estimator of diffusion entropy is proposed to detect quantitatively scalings in short time series. The effectiveness is verified by detecting successfully scaling properties for a large number of artificial fractional Brownian motions. Calculations show that this method can give reliable scalings for short time series with length \( \sim 10^7 \). It is also used to detect scalings in the Shanghai Stock Index, five stock catalogs, and a total of 134 stocks collected from the Shanghai Stock Exchange Market. The scaling exponent for each catalog is significantly larger compared with that for the stocks included in the catalog. Selecting a window with size 650, the evolution of scaling for the Shanghai Stock Index is obtained by the window’s sliding along the series. Global patterns in the evolutionary process are captured from the smoothed evolutionary curve. By comparing the patterns with the important event list in the history of the considered stock market, the evolution of scaling is matched with the stock index series. We can find that the important events fit very well with global transitions of the scaling behaviors.

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I. INTRODUCTION

Scale invariance widely exists in complex systems in diverse research fields [1]. Consider a stochastic trajectory \( X(t) \), whose statistical properties are described by the probability distribution function (PDF) of the displacements \( p(x,t) \). The stochastic process represented by \( X(t) \) behaves scale invariant, provided the PDF satisfies

\[
p(x,t) = \frac{1}{p^\delta} F \left( \frac{x}{p^\delta} \right),
\]

where \( \delta \) is the scaling exponent. For ordinary statistical mechanics, the function \( F(\cdot) \) has a Gaussian form and \( \delta = 0.5 \) [2,3]. By using the scaling exponents one can describe quantitatively the deviations from ordinary mechanics, and consequently assess the real physical nature of a phenomenon. But evaluation of scaling exponents meets several challenges.

How to evaluate a reliable scaling is not a trivial task. Variance-based methods are used widely in literature to calculate the scaling exponents [4], in which there exists an assumption of the time dependence of the variance \( \text{Var}[X(t)] \) with the scaling exponent \( \delta \), namely, \( \text{Var}[X(t)] \sim t^{2\delta} \). For fractional Brownian motions it is valid, but there are scaling processes such as Lévy flights, for which the second moment diverges, or Lévy walks, for which the second moment satisfies a scaling relation \( \text{Var}[X(t)] \sim t^{2H} \) with \( H \neq \delta \) (i.e., the relationship is violated) [5]. Several efforts have been made to develop complementary methods to evaluate reliable scaling exponents [6,7]. To cite an example, from the PDF one can calculate the Shannon entropy, \( S(t) = - \int p(x,t) \log p(x,t) dx \), which is originally identified as diffusion entropy by Scafetta et al. [6]. It is proved that the diffusion entropy can provide simultaneously reliable values of \( \delta \) for fractional Brownian motions and Lévy processes [8].

For a real-world stochastic process, the PDF is generally not known. One can count how often the value \( x \) appears in the trajectory \( X(t) \). Denoting the number with \( n(x) \), the PDF can be estimated with the relative frequency \( \frac{n(x)}{N} \). \( N \) is the total size of the data set. In many situations only small data sets from which to infer PDF are available. What is more, for a stochastic process with a large amount of data, there exist generally structural breaks in the trajectory due to emergent strikes from environments and/or the system’s transition to a contrasting dynamical regime. To cite examples, a stock market is shocked frequently by currency and tax policies, and before and after an earthquake the earth motion may stay in different dynamical regimes. We should separate the data into segments to detect behaviors at different structural patterns. A small value of \( N \) may induce large statistical fluctuations or even bias to physical quantities, such as PDF, entropy, and moments. In a recent paper, Bonachela et al. recall the search for improved estimators of entropy for small data sets [9]. They propose also a balanced estimator that performs well when the data sets are small and \( p(x,t) \) are not close to zero.

Stimulated by the two mentioned efforts, in the present paper a concept is introduced, called balanced estimator of diffusion entropy (BEDE), in which the balanced estimator of entropy is used to replace the original form in the diffusion entropy. This concept is used to find scalings and structural breaks in artificial and empirical series. Firstly, we review briefly the concepts of diffusion entropy and balanced estimator, and introduce consequently the concept of BEDE. Secondly, the effectiveness of the BEDE in detecting scalings in short time series are verified by means of a large amount of fractional Brownian motions. Finally, we detect the scalings and structural breaks in the stock price series of the Shanghai Stock Market.

II. METHODS AND MATERIALS

A. Diffusion entropy

We review briefly the concept of diffusion entropy [6,8]. Let us consider a one-dimensional stationary time series,

\[\xi_1, \xi_2, \ldots, \xi_N.\]
$N$ is the length of the series. All the possible segments with length $s$ read

$$X_i = [\xi_i, \xi_{i+1}, \ldots, \xi_{i+s-1}], \quad i = 1,2,\ldots,N-s+1. \quad (3)$$

Now we regard the length $s$ as time, the vector $X_i$ can be regarded as a stochastic trajectory of a particle starting from its initial position $X_i(0) = 0$. By this way, the time series (2) is mapped to an ensemble containing $N-s+1$ realizations of a stochastic process. The displacements read

$$x_i(s) = \sum_{j=1}^{s} X_i(j) = \sum_{j=i}^{i+s-1} \xi_j, \quad i = 1,2,\ldots,N-s+1. \quad (4)$$

One can divide the displacement interval where the particles appear into $M(s)$ bins, and reckon the number of particles occurring in each bin at time $s$. We denote the numbers with $n(j,s), j = 1,2,\ldots,M(s)$. The PDF can be naïvely approximated by the relative frequency

$$p(j,s) \sim \hat{p}(j,s) = \frac{n(j,s)}{N-s+1}, \quad j = 1,2,\ldots,M(s). \quad (5)$$

The entropy of the diffusion process is consequently determined, which reads

$$S_{DE}(s) \sim S_{DE}^{\text{naïve}}(s) = -\sum_{j=1}^{M(s)} \hat{p}(j,s) \ln[\hat{p}(j,s)]. \quad (6)$$

This entropy is based upon the diffusion process constructed from the original series (2), for this reason is called diffusion entropy (DE) [8].

The size of the bins, $e(s)$, is usually chosen to a fraction of the square root of the variance of the original series (2), which is independent of $s$.

Now we assume the time series behaves scale invariant, namely, $p(j,s)$ obeys the relation (1)

$$p(j,s) = \frac{1}{s^\beta} F\left(\frac{x_{\text{min}}(s) + (j-0.5)e(s)}{s}\right), \quad j = 1,2,\ldots,M(s), \quad (7)$$

where $x_{\text{min}}(s)$ is the smallest value of displacement, i.e., $x_{\text{min}}(s) = \min[x_1(s),x_2(s),\ldots,x_n(s)]$. Let us plug Eq. (7) into Eq. (6). A simple computation leads to

$$S_{DE}(s) = A + \delta \ln(s), \quad (8)$$

where $A = \int_{0}^{\infty} dy F(y) \ln[F(y)]$.

The simple relation of Eq. (8) can be used to detect scalings in time series. It is an effective tool yielding the correct scaling in both the Gauss and the Lévy statistics. For this reason, it attracts special attention from diverse research fields [10].

### B. Balanced estimator for diffusion entropy

From the relation Eq. (5) we have the ensemble average

$$\langle \hat{p}(j,s) \rangle = \frac{n(j,s)}{N-s+1} = p(j,s). \quad (9)$$

The frequencies $\hat{p}(j,s)$ are unbiased estimators of the probabilities $p(j,s)$. Approximating the probabilities with the frequencies $\hat{p}(j,s)$ may lead to certain statistical errors (variance), but does not induce any systematic error (bias).

However, the approximation may introduce a bias to the entropy. Defining an error variable, $\mu(j,s) = \frac{p(j,s) - \hat{p}(j,s)}{p(j,s)}$, after a straightforward algebraic we have [11]

$$S_{DE}(s) = S_{DE}^{\text{naïve}} + \frac{M(s) - 1}{2(N-s+1)} + O[M(s)]. \quad (10)$$

For finite $N-s+1$, the leading order of error, $\frac{M(s)-1}{2(N-s+1)}$, is a significant error. It vanishes only as $(N-s) \to \infty$. Consequently, $S_{DE}^{\text{naïve}}(s)$ is a biased estimator of $S_{DE}(s)$ (i.e., it deviates from the true entropy not only statistically but also systematically).

We employ the solution proposed in Ref. [9] to solve this problem, namely, the balanced estimator of entropy. The basic idea is to reduce the bias or the variance as much as possible. Defining $S_{DE}[p(j,s)] \equiv -p(j,s) \ln[p(j,s)]$, we have $S_{DE}(s) = \sum_{j=1}^{M(s)} S_{DE}[p(j,s)]$. We want to find an estimator,

$$\hat{S}_{DE}(s) \equiv \sum_{j=1}^{M(s)} \hat{S}_{DE}[n(j,s)], \quad (11)$$

to minimize the bias,

$$\Delta_{\text{bias}}^2(s) = (\langle \hat{S}_{DE}(s) \rangle - S_{DE}(s))^2 \quad (12)$$

or the mean squared deviation

$$\Delta_{\text{stat}}^2(s) = \langle (\hat{S}_{DE}(s) - \langle \hat{S}_{DE}(s) \rangle)^2 \rangle \quad (13)$$

or a combination of both as much as possible.

A reasonable assumption is that we can ignore the correlations between the elements of the distribution, $n(j,s), j = 1,2,\ldots,M(s)$. The task is simplified to minimize simultaneously the bias and the variance for each summand,

$$\Delta_{\text{bias}}^2[p(j,s)] = (\langle \hat{S}_{DE}[n(j,s)] \rangle - S_{DE}[p(j,s)])^2, \quad (14)$$

and

$$\Delta_{\text{stat}}^2[p(j,s)] = \langle (\hat{S}_{DE}[n(j,s)] - \langle \hat{S}_{DE}[n(j,s)] \rangle)^2 \rangle, \quad (15)$$

where the probability $p(j,s) \in [0,1]$, and $n(j,s) \in \{0,1,2,\ldots,N-s+1\}$ is binomially distributed. To balance the errors, we minimize the average error over the whole range of $p(j,s) \in [0,1]$,

$$\Delta^2(j,s) = \int_{0}^{1} dp[j,s] w[p(j,s)] \left[ \Delta_{\text{bias}}^2(j,s) + \Delta_{\text{stat}}^2(j,s) \right]. \quad (16)$$

where $w[p(j,s)]$ is a suitable weight function on which that specific problem depends. Without extra knowledge of the probability values, one can consider a simple case of $w[p(j,s)] = 1$. Inserting Eqs. (14) and (15) into Eq. (16), the average error is given by

$$\Delta^2(j,s) = \int_{0}^{1} dp[j,s] \left\{ \sum_{n(j,s)=0}^{N-s+1} P_{n(j,s)} [p(j,s)] \hat{S}_{DE}[n(j,s)] \right\} \right. \left. + S_{DE}^2[p(j,s)] - 2S_{DE}[p(j,s)] \right. \left. \times \left( \sum_{n(j,s)=0}^{N-s+1} P_{n(j,s)} [p(j,s)] \hat{S}_{DE}[n(j,s)] \right) \right); \quad (17)
where $P_{n(j,s)}[p(j,s)]$ is the binomial distribution,

$$
P_{n(j,s)}[p(j,s)] = \frac{[N-s+1]!}{n(j,s)! [N-s+1-n(j,s)]!} \times [p(j,s)]^{n(j,s)} [1-p(j,s)]^{N-s+1-n(j,s)}.
$$

(18)

The expected numbers $\hat{S}_{DE}[n(j,s)]$, $j = 1, 2, \ldots, N - s + 1$ should lead to the minima of the average error. The necessary condition is that all the partial derivatives vanish,

$$
\frac{\partial \Delta^2(j,s)}{\partial \hat{S}_{DE}[n(j,s)]} = 0, \quad n(j,s) = 1, 2, \ldots, N - s + 1.
$$

(19)
i.e.,

$$
\int_0^1 dp(j,s) P_{n(j,s)}(\hat{S}_{DE} - S_{DE}) = 0.
$$

(20)

Hence, the balanced estimator can be obtained as follows,

$$
\hat{S}_{DE}[n(j,s)] = \int_0^1 dp(j,s) P_{n(j,s)} S_{DE} = (N - s + 2) \int_0^1 dp(j,s) P_{n(j,s)} [-p(j,s) \ln[p(j,s)]]
$$

$$
= -\frac{(N - s + 2)!}{n(j,s)! [N - s + 1 - n(j,s)]!} \int_0^1 [1 - p(j,s)]^{N-s+1-n(j,s)} p(j,s)^{n(j,s)} \left( \lim_{z \rightarrow 1} \int_0^1 dp(j,s) \left( \frac{dp(j,s)^z}{dz} \right) dp(j,s) \right)
$$

$$
= -\frac{(N - s + 2)!}{n(j,s)! [N - s + 1 - n(j,s)]!} \lim_{z \rightarrow 1} \int_0^1 \left[ \frac{[N-s+1-n(j,s)]!}{[n(j,s)+1+z][n(j,s)+2+z]} \right] [n(j,s)+2][n(j,s)+3] \cdots [N-s+3] \sum_{k=n(j,s)+2}^{N-s+3} \frac{1}{k}
$$

$$
= \frac{n(j,s)+1}{N - s + 3} \sum_{k=n(j,s)+2}^{N-s+3} \frac{1}{k},
$$

(21)

and

$$
\hat{S}_{DE}(s) = \frac{1}{N - s + 3} \sum_{j=1}^{M(s)} [n(j,s)+1] \sum_{k=n(j,s)+2}^{N-s+3} \frac{1}{k},
$$

(22)

called Balanced Estimator of Diffusion Entropy (BEDE) in this paper.

C. Materials

1. Fractional Brownian motions

Fractional Brownian motions (fBm) [12] are used to verify the effectiveness of BEDE in detecting scalings in short time series. A fBm is a continuous-time Gaussian process depending on the Hurst parameter $0 \leq H \leq 1$. The fBm is self-similar in distribution and the variance of the increments is given by $\sim |s-t|^H$. The program wfbm.m in Matlab® is used to generate the fBm series.

2. Shanghai Stock Exchange indices

Empirical data are collected from the Shanghai Stock Exchange (SSE) [13], the world’s fifth largest stock market by market capitalization at US$2.7 trillion as of December 2010. The current exchange was established on November 26, 1990 and was in operation on December 19 of the same year. We collect a total of 134 closing price series starting from the end of the year 1995 to the end of June, 2010, in which the number of stocks distributed in the categories of industry, business, real estate, public utility, and comprehension are 64, 27, 12, 19, and 19, respectively. We consider also the stock price indices of the five categories from December 6, 1994 to June 30, 2010. The SSE index series starts from December 19, 1990 and ends at June 30, 2010.

For a closing price series, $p(t)$, one can construct the corresponding return series

$$
r(t) = \frac{\log[p(t + \Delta t)]}{\log[p(t)]}.
$$

(23)

If $\Delta t > 1$, the resulting return series is generally nonstationary, which will violate the stationary assumption in the diffusion entropy approach. In the following calculations, $\Delta t$ is selected to be 1.

III. RESULTS

Figure 1 presents several typical examples of the comparison between BEDE and DE. For the three generated fBm series [Figs. 1(a)–1(c)], with $H = 0.7$ and lengths 650,650,4000, respectively, in a wide range of $s$, the BEDEs obey almost perfect linear relations versus $\ln(s)$ as shown in Figs. 1(d)–1(f) (i.e., the scalings are all perfectly rendered out). The estimated values of the scalings (the slope of the BEDE curve and denoted with $\delta_B$) are 0.70,0.67, and 0.71, which can be
FIG. 1. (Color online) Typical examples of the comparison between BEDE and DE. (a)–(c) Generated fBm motions with \( H = 0.7 \) and lengths 650, 650, 4000, respectively. (d)–(f) are the corresponding BEDE and DE curves. In a wide range of \( s \), BEDEs obey almost perfect linear relations versus ln\( (s) \). Estimated values of the scalings (the slope of the BEDE curve, denoted with \( \delta_B \)) are \( 0.70, 0.67, \) and \( 0.71 \). With the increase of time, the DE curves tend to bend down and the deviations from the linear relations of BEDE versus ln\( (s) \) become more and more significant. With the increase of length (e.g., 4000) the DE curve is corrected significantly to be much closer to the BEDE curve in a wider range of ln\( (s) \), as shown in (f). Panels (g)–(i) are three fBm series with length 650 and \( H = 0.3, 0.5, \) and 0.9, respectively. (j)–(l) are the corresponding BEDE and DE curves for the three series. Incorporating with panels (d)–(e), one can find that with the increase of \( H \), the bias becomes much more significant. For small values of \( H \) (e.g., \( H = 0.3 \)) the bias is negligible.

FIG. 2. (Color online) Confidence of BEDE-based scaling estimation. (a) \( 10^4 \) series with \( N = 650 \) and \( H = 0.5, 0.6, \) and 0.7 are generated, respectively. The scaling estimations distribute normally and center at the expected values of \( \delta_B = 0.50, 0.6, \) and 0.7. (b) The relation of certainty level versus series length at a specified confidence interval \( \Delta H = 0.08 \). Hence, for a series with short length and large value of \( H \), the calculated values of DE have unacceptable errors due to bias. The curve of DE versus ln\( (s) \) can not detect correctly the scaling at all, while the BEDE can give perfect estimations of entropy even for considerably large values of \( s \), namely, a small set of data \( (N - s + 1 \) records).

For a specific value of \( H \), one can generate a large number of fBm series. It is found that the scaling estimations distribute normally, as shown in Fig. 2(a) with a typical example, in which totally \( 10^4 \) series with \( N = 650 \) and \( H = 0.5, 0.6, \) and 0.7 are used. By specifying a confidence interval \( [H - \Delta H, H + \Delta H] \) the corresponding level of certainty \( p_{\text{conf}} \) is determined so that \( p_{\text{conf}} \times N_{\text{conf}} \) estimations occur in the confidence interval. \( N_{\text{conf}} \) is the total number of the generated fBm series. Figure 2(b) shows the relation of the certainty level \( p_{\text{conf}} \) versus the series length \( N \). At the beginning, with the increase of \( N \), \( p_{\text{conf}} \) increases rapidly, while when \( N \) becomes large \( p_{\text{conf}} \) tends to saturate to a high value. Accordingly, we select \( N = 650 \) in the following calculations in detecting local scaling behaviors of the stock series in SSE stock market.

We calculate the BEDEs for the Shanghai Stock Exchange (SSE) index, and the indices of the five catalogs including industry, business, real estate, public utility, and comprehension. In a wide range of \( s \), the relations of BEDE versus ln\( (s) \) obey a linear law. Hence, there exist almost perfect self-similarities and the scaling exponents are \( \delta_{\text{SSE}} = 0.58, \delta_{\text{ind}} = 0.55, \delta_{\text{pub}} = 0.58, \delta_{\text{rea}} = 0.57, \delta_{\text{bus}} = 0.56 \), and \( \delta_{\text{com}} = 0.54 \), respectively [see Fig. 3(a)]. The scaling estimations for the selected 134 stocks are also calculated, as shown in Fig. 3(b). The scaling exponent for each catalog is significantly larger compared with that of the stocks included in the corresponding catalog. Though each specific stock is almost not predictable, the

regarded as the same as the expected value of \( H = 0.7 \). However, for the short series in Figs. 1(a) and 1(b), with the increase of time, the DE curves tend to bend down and the deviations from the linear relations of BEDE versus ln\( (s) \) become more and more significant. With the increase of length [e.g., 4000 in Fig. 1(c)] the DE curve is corrected significantly to be much closer to the BEDE curve in a wider range of ln\( (s) \), but at the region with larger values of \( s \) it still bends down with an unacceptable bias.

Figures 1(g)–1(i) present three generated fBm series with length 650 and \( H = 0.3, 0.5, \) and 0.9, respectively. The corresponding BEDE and DE curves are shown in Figs. 1(j)–1(k). Incorporating with the curves in Figs. 1(d)–1(e), we can find that with the increase of \( H \), the bias becomes more and more significant. For time series with small values of \( H \), there is not a distinctive difference between BEDE and DE curves. A large number of calculations verify the robustness of this conclusion.
In a wide range of $s$, the relations of BEDE versus $\ln(s)$ obey a linear law. The scaling exponents are $\delta_{\text{SSE}} = 0.58$, $\delta_{\text{Ind}} = 0.55$, $\delta_{\text{Re}} = 0.58$, $\delta_{\text{Pu}} = 0.57$, $\delta_{\text{Com}} = 0.56$, and $\delta_{\text{om}} = 0.54$, respectively. (b) Scaling estimations for the selected 134 stocks. Scaling exponent for each catalog is significantly larger compared with that of the stocks included in the corresponding catalog. (c) Comparison of scaling estimations detected by DE (denoted with $\delta_D$) and BEDE (denoted with $\delta_B$). Most values of $\delta_B - \delta_D$ are in the interval of $[-0.05, 0.05]$.

For the SSE series, $\{r_{\text{SSE}}(1), r_{\text{SSE}}(2), \ldots, r_{\text{SSE}}(N)\}$, one can calculate the scaling exponents for all the segments of $\{r_{\text{SSE}}(t - s + 1), r_{\text{SSE}}(t - s + 2), \ldots, r_{\text{SSE}}(t)\}, t = s, s + 1, \ldots, N$, denoted with $\delta_{\text{SSE}}(t = \Delta \tau)$, which are employed in the present work to represent the local scalings of the SSE series, as shown in Fig. 4(a). The value of $s$ is chosen to be 650. In the more than ten years duration the value of $\delta_B$ distributes in a wide interval from 0.42 to 0.71. The scaling for the total series is 0.58. Assuming a structural break occurs at time $t$, only when the segment covers a certain number of data after the time $t$, the contribution from the break’s occurrence becomes significant and detectable. We introduce the parameter $\Delta \tau$ to describe this kind of delay effect.

From Figs. 1 and 2 one can find that to keep the confidence level and confidence interval acceptable (e.g., confidence level $>0.85$ and confidence interval $<0.08$), for the series with $0.4 \leq H \leq 0.7$ the length should be larger than 650. Much smaller value of the window size may induce an unacceptably low confidence level, while much longer windows may lead to loss of local information. This is the reason we select 650 as the window size. Actually, we have also calculated the results with much larger values of the window size and the shapes of the evolutionary curves are similar with that reported in Fig. 4(a).

It is reasonable to believe that important events, such as policies and/or emergencies, may lead to speedy transitions...
of a stock market from lower (higher) to higher (lower) predictability. The effect should persist for a long time. Hence, in the present paper we determine the delay $\Delta \tau$ by means of comparison of the patterns in $\delta_B(t)$ with that in the important events list.

To filter out local fluctuations, the evolutionary curve of $\delta_B(t)$ is smoothed by replacing each value with the average of its neighbors and itself, namely,

$$\delta_B^*(t) = \frac{\sum_{t'=t-m}^{t+m} \delta_B(t')}{\sum_{t'=t-m}^{t+m} \omega(t')} ,$$

(24)

where $\delta_B(t') = 0$ for $t' < \min(t)$ and $t' > \max(t)$, and $\omega(t') = 1$ if $\min(t) \leq t' \leq \max(t)$ and 0 otherwise. From the smoothing curve one can find that though there exist rich fine structures with locally abrupt changes, there are globally four peaks covering 427,613,1782, and 1313 data points, namely, persisting roughly 20, 30, 86, and 63 months, respectively.

Let us recall the important events occurring in the history of the SSE market [14]. Three important events are marked with magenta lines. The first event, $T_1$, corresponds to the bull market in the duration from July 29, 1994 to September 13, 1994. Before this bull market, the market suffered from a 17-month duration of decrease. The China Securities Regulatory Commission (CSRC) issues three special policy regulation items to bail out the stock market. Accordingly, the SSE index increases rapidly from 325 to 1052 within one and a half months (reaches the record at September 13, 1994).

The second event, from $T_2a$ to $T_2b$, is the bull market in the duration from January 19, 1996 to May 12, 1997, in which the stock index rises up to 1464 from 512. At the time speculating blue chip stocks tends to dominate the investment concept. The Shenzhen Development Bank and Sichuan Changhong become successively the leading stocks in the Shenzhen Stock Exchange market and the Shanghai Stock Exchange market, respectively. Stimulated by the two stocks, the SSE market becomes highly active and after the National day of China, the prices for almost all stocks increase rapidly. The CSRC issues successively some policy regulation items to cool down the stock market and expounds in detail the irrational state of the stock market.

Starting from June 13, 2001, the day a local maxima occurs at level 2242, a decreasing process persists for about 48 months, during which a bouncing maxima occurs at level 1778 at April 6, 2004. At May 9, 2005 the reform of the shareholder structure of listed companies (RSSLC) is conducted, which induces a persistent increase of the SSE index for about 24 months. This event is denoted with $T_3$. Then the persistent increase is disturbed to a chaotic state by the escalation of stamp tax at May 30, 2007.

Consequently, the delay is determined to be $\Delta \tau = 60$. By this way the stock series and the $\delta_B$ evolutionary series are matched along time, as shown in Fig. 4. The three events are all very close to the local minima or maxima of valleys or peaks in the smoothing curve, respectively, where transitions of global behaviors occur. On the left of $T1,T2b$, and $T3$, $\delta_B(t)$ decreases persistently, while on the right of them $\delta_B(t)$ increases persistently. After a monotonically increase on the left of $T2a$, $\delta_B(t)$ comes to a short-time stable state, then decreases rapidly.

It should be noted that for short time series, correction of bias is the key step to obtain correct scalings. From Fig. 4(b) one can find that scalings detected by BEDE are generally larger than that by DE. Many values of $\delta_B - \delta_B^*$ are larger than 0.1 and even reach 0.2, which can not be explained by low resolutions. We show several typical examples in Figs. 4(c)–4(f) in which large bias in DE curves are corrected in BEDE curves.

IV. CONCLUSION

Scaling invariance holds in a large amount of complex systems, but the evaluation of scaling is still a challenging task. Theoretically, variance-based methods can not detect correctly the scalings for Lévy processes. Empirically, time series are usually not long enough to derive a reliable scaling exponent. What is more, in a long time series there are usually structural breaks. In literature, there are two stimulating efforts in solving the problems: diffusion entropy is developed to detect reliable scaling exponents for long time series and balanced estimator of entropy is proposed to correct bias in approximation of entropy for short time series. In the present paper, balanced estimation of entropy for short time series is introduced to the diffusion entropy to find reliable scalings embedded in short time series.

This method can give reliable scalings even for short time series with length $\sim 10^2$. It is used to detect the scalings embedded in a total of 134 stocks in the SSE market. The scaling exponent for each catalog is significantly larger compared with that for the specific stocks included in the catalog.

We detect also the local scalings in the SSE index series. The scalings vary in a large interval from 0.42 to 0.71. By comparing the important events list in the history of the Shanghai Stock Exchange market and the patterns of the smoothing for the scaling evolutionary curve, the scaling evolutionary curve is matched with the SSE index series with a delay of $\Delta \tau = 60$. The important events can fit with the global transitions in the scaling evolution very well.

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