Analysis on Cavitation Characteristics of Flow in a Francis Turbine with Different Content of Non-Condensable Gas

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Abstract. Cavitation causes serious damages to Francis turbine, e.g., noise and vibration. Its mechanism is complex and may be affected by many factors. The present paper compares cavitation behavior of flow in a Francis turbine with different content of non-condensable gas (NCG) concluded from experiment and numerical simulations. The experimental results show small difference in characteristics of cavitation with different content of non-condensable gas, while numerical simulation shows larger difference. It can thus be concluded that present simulation over-predict this difference.

Keywords. Francis Turbine, Cavitation, Non-Condensable Gas (NCG).

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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Guide vane</td>
<td>[mm]</td>
</tr>
<tr>
<td>F_vap</td>
<td>Coefficient of evaporation, 50</td>
<td>[-]</td>
</tr>
<tr>
<td>F_con</td>
<td>Coefficient of condensation, 1</td>
<td>[-]</td>
</tr>
<tr>
<td>n_11</td>
<td>Unit rotation speed, [rpm]</td>
<td></td>
</tr>
<tr>
<td>NCG</td>
<td>Non-condensable gas, [mg/L]</td>
<td></td>
</tr>
<tr>
<td>Q_11</td>
<td>Unit flow rate, [L/s]</td>
<td></td>
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<tr>
<td>p</td>
<td>Local static pressure, [Pa]</td>
<td></td>
</tr>
<tr>
<td>R_B</td>
<td>The radius of the nucleation sites, [m]</td>
<td></td>
</tr>
<tr>
<td>\alpha</td>
<td>Volume fraction, [-]</td>
<td></td>
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<tr>
<td>\alpha_g</td>
<td>Non-condensable gas void fraction, [-]</td>
<td></td>
</tr>
<tr>
<td>\mu</td>
<td>Kinematics viscosity, [Pa s]</td>
<td></td>
</tr>
<tr>
<td>\rho</td>
<td>Fluid density, [Kgm^{-3}]</td>
<td></td>
</tr>
<tr>
<td>\sigma_i</td>
<td>Incipience Thoma number, [-]</td>
<td></td>
</tr>
<tr>
<td>\sigma_c</td>
<td>Critical Thoma number, [-]</td>
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Subscripts

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>i, j</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>v</td>
<td>Water vapor</td>
</tr>
<tr>
<td>l</td>
<td>Water</td>
</tr>
<tr>
<td>m</td>
<td>Mixture</td>
</tr>
<tr>
<td>exp</td>
<td>Experiment</td>
</tr>
<tr>
<td>cal</td>
<td>Calculation</td>
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1 Introduction

Cavitation and bubble dynamics have been the subject of extensive research since the early works of Besant in 1859 ([11]) and Lord Rayleigh in 1917 ([12]). Various physical aspects ([3–6]) of bubble dynamics have been considered, such as inertia, interface dynamics, gas diffusion, heat transfer, etc.

In practice, the main mechanism of cavitation inception is treated as the explosive growth of small bubbles comprising of non-condensable gases and vapor in the water. This non-condensable gas is small bubbles, filled with air (or other gases) and vapor. Their diameter can vary from a few microns to nearly visible bubbles of the order of 1 mm.

Non-condensable gas amount is an important factor to cavitation inception and development. Avellan et al. ([7]) demonstrates that the hydrodynamic characteristic for the Francis turbine model are changed by the presence of nuclei. Briancon-Marjollet and Frechou ([8]) depict the different Thoma numbers on propeller in different amounts of non-condensable gas, which is shown in Figure 1. Furthermore, propeller’s efficiency and advanced coefficient is affected by the presence of nuclei at the same advanced coefficient in Figure 2 ([9]). However, Gindroz ([10]) demonstrates effect of the amount of the nuclei to the incipient Thoma number is smaller when the cavitation type is sheet cavitation.

2 Experiment

The experiment has been carried out on the Francis turbine test station, as shown in Figure 3. The comprehensive testing error of efficiency of the test station is smaller than ±0.25%. The water head for this Francis turbine is 30 m, and the unite rotation speed is 60 r/min. The accuracy of the each experimental facility is shown as follows:
Electromagnetic flowmeter (MS900F): Its accuracy is ±0.2%. Two tandem electromagnetic flowmeters are installed. The 5 m³ graduated cylinder’s accuracy is ±0.05%.

Torque for the kinematics moment (1110-A0-1K): HBM-Z6FC3 sensor for load’s accuracy is ±0.02%Fs. The arm length of the measuring power machine is 1,300 mm and the tolerance is 0.1 mm. The measurement weight range is 0–200 kg and the accuracy is ±0.005%.

Torque for the friction moment (1110-A0-1K): The sensor’s stated accuracy is ±0.2%Fs. The measurement weight range is 1 kg and the accuracy is ±0.05%.

Head (3051CD4A22A1A): The measuring apparatus is differential pressure type head gauge ROSEMOUNT-3051. Its stated accuracy is ±0.04%Fs.

Rotation speed (MP981): The measuring apparatus is OMRON-E6B2 encode, which is adjusted by the velocity feedback method. Its stated accuracy is smaller than ±0.1%.

Water pressure of tail water (3051TA1A2B21A): The measuring apparatus’ range is 0–200 kPa. Its accuracy is 0.075%.

Dissolved oxygen apparatus: The measuring apparatus’ range is 0–60 ppm. Its accuracy is ±0.01.

The experimental procedure of cavitation test is: at a given rotational speed, the flow rate, measured by the flowmeter, is set to the operating value using the motorized control valve. The outlet pressure drop is realized by the liquid vacuum pump. The gas amount is achieved by converting the obtained content of dissolved oxygen. For certain operating values of flow rates and outlet pressure, specific types of cavitation are observed. When cavitation bubbles appear on three turbine blades simultaneously, the corresponding Thoma number is named incipient Thoma number. Furthermore, an efficiency decrease of 0.5% with very low outlet pressure values was achieved. This point is characterized with the so-called critical Thoma number.

3 Mathematic Models

3.1 Mixture Mass, Momentum

Since a basic assumption to the model is that all phases share the same velocity, the governing equations for mass and momentum of the mixture are

\[ \frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial x_j} (\rho_m u_j) = 0, \]  
\[ \frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial x_j} (\rho_m u_j u_i) = -\frac{\partial}{\partial x_i} \tau_{ij} - \frac{\partial p}{\partial x_i}, \]  

where the density is defined through volume fractions as \( \rho_m = \alpha_l \rho_l + \alpha_g \rho_g \) and the velocity field, \( u_j \), and is calculated from the momentum equations, \( \tau_{ij} \) is the deformation tensor and is further simplified by the assumption of Newtonian fluid behavior. The flows in the devices presented in this paper are turbulent and require turbulence models to estimate the enhanced mixing effect on the flow.
3.2 RNG k-ε Model

The $k$ and $\varepsilon$ equations are written as

$$
\frac{\rho_m}{D_k} \frac{D k}{Dt} = \frac{\partial}{\partial x_j} \left( \alpha_k \mu_{\text{eff}} \frac{\partial k}{\partial x_j} \right) + 2 \mu_t \langle S_j \rangle \frac{\partial (u_i)}{\partial x_j} - \rho_m \varepsilon, \tag{3}
$$

$$
\frac{\rho_m}{D\varepsilon} \frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_j} \left( \alpha_\varepsilon \mu_{\text{eff}} \frac{\partial \varepsilon}{\partial x_j} \right) + 2 C_{1\varepsilon} \frac{\varepsilon}{k} \varepsilon \langle S_j \rangle \frac{\partial (u_i)}{\partial x_j}
- C_{2\varepsilon} \rho_m \frac{\varepsilon^2}{k} - R. \tag{4}
$$

The effective viscosity is written as $\mu_{\text{eff}} = \mu + \mu_t$.

$$
\tilde{\nu}_{ij} = \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right).
$$

The additional term $R$ is written as

$$
R = C_{\mu} \rho \eta^3 (1 - \eta/\eta_0) \frac{\varepsilon^2}{1 + \beta \eta^3 \frac{k}{\varepsilon}}.
$$

Empirical coefficients are chosen as $\eta = S_k^{1/2}, \eta_0 = 4.38, C_{\mu} = 0.0845, \beta = 0.012, C_{1\varepsilon} = 1.42, C_{2\varepsilon} = 1.68, \alpha_k = 1.0$ and $\alpha_\varepsilon = 0.769$.

3.3 Cavitation Model

Most cavitation models are based on the utilization of Rayleigh Plesset model ([11, 12]). Generally, cavitation is treated without thermal phase change. The cavitation process is typically so fast, the assumption of thermal equilibrium at the interface remains valid through the process. In the simplest forms of cavitation models, mass transfer is driven by purely mechanical effects, namely liquid-vapor pressure differences, rather than thermal effects. So the equations can be written as:

If $p > p_v$:

$$
\dot{m} = F_{\text{con}} \frac{3 \alpha_v \rho_v}{R_B} \sqrt{\frac{2}{3} \frac{|p_v - p|}{\rho_l}} \text{sgn}(p_v - p). \tag{5}
$$

If $p < p_v$:

$$
\dot{m} = F_{\text{vap}} \frac{3 \alpha_v (1 - \alpha_v) \rho_v}{R_B} \sqrt{\frac{2}{3} \frac{|p_v - p|}{\rho_l}} \text{sgn}(p_v - p). \tag{6}
$$

The void fraction of the nuclei is used in the simulation for the prediction of cavitation inception. However, this value is not available through our experiments. The amount of non-condensable gas has been used instead. This might cause deviation in the simulation result.

4 Results and Discussion

The hydroturbine’s full passage calculation zone is shown in Figure 5, which is divided into four subdomains: spiral case, stay vane, guide vane, runner and draft tube. The draft tube is divided into cone, elbow and diffuser. The hydroturbine has 17 runner blades and 24 guide vanes.

**Inlet boundary condition:** The total pressure is specified at the inlet boundary condition. In the computation, the default inlet turbulence intensity, $I$ is selected, and the value is set as $I = 0.05$, which is an approximate value for the internal pipe flow. The inlet turbulence energy and the turbulence dissipation (RNG k-ε model) could be calculated.
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<table>
<thead>
<tr>
<th>Condition</th>
<th>NCG (mg/L)</th>
<th>a (mm)</th>
<th>( n_{11} ) (rpm)</th>
<th>( Q_{11} ) (L/s)</th>
<th>( \sigma_{t-cal.} )</th>
<th>( \sigma_{c-cal.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20.3</td>
<td>26</td>
<td>58.89</td>
<td>546</td>
<td>0.07</td>
<td>0.4</td>
</tr>
<tr>
<td>B</td>
<td>2.36</td>
<td>26</td>
<td>58.89</td>
<td>546</td>
<td>0.02</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2. Calculation working conditions.

Figure 5. The structure of Francis turbine.

Outlet boundary condition: At the pressure outlet, the pressure is assigned according to the Thoma number, vapor pressure and suction height.

Boundary conditions near solid walls: For the fully developed turbulent flow, the boundary conditions near solid walls are implemented using wall functions and no-slip condition is applied on the wall surface.

Numerical treatment: In the simulation, the second order upwind scheme is used for the discretization of the convective term, and the second order central scheme for the discretization of the diffusion term and other source terms. The implicit method is used to solve the incompressible flow in the present simulation. The discrete momentum equations and the continuity equation for the complete flow field are solved together with iteration and corrections.

The working conditions for the calculation are shown in Table 2. The effect of non-condensable gas amount on cavitation development of the Francis turbine is strong. The difference of incipient Thoma number comes up to 71% between A and B in Figure 6. However, the difference is only 1% in the experiment results.

Pressure and vapor distribution on the runner are shown in Figure 7. On the incipient of working condition A, when pressure is lower than saturated vapor pressure, the vapor is generated on the blade rim. Cavitation bubbles appear on 3 to 4 blades (the white color is the cavitation zone). The vapor volume fraction is very small.

Figure 8 shows the vapor distribution on the blade, and vapor covers every blade rim on the critical Thoma number. The white bubbles on the blade (10% water vapor volume fraction) are shown on the right of Figure 8. On working condition A, the efficiency decreases 0.5% when the cavitation is critical Thoma number.

However, pressure and vapor distribution are similar between working conditions A and B in Figures 9 and 10. However, the incipient Thoma number and critical Thoma number are different because of the non-condensable gas amount. The incipient Thoma number is smaller on working condition B than on working condition A.

By comparing the experimental results and the results from simulations, it can be concluded that the simulations predict significantly different cavitation inception with different amount of non-condensable gas regarding this Francis turbine, which deviate from experiments.

5 Conclusions

The Thoma coefficient is obtained with numerical simulation, and the result is compared with experiment. It is shown that the numerical simulation cannot correctly predict the change of cavitation inception with varying amount of non-condensable gas. The change in the amount of non-condensable gas may cause change in the amount of nuclei. Lack of information on nuclei may be the reason of this discrepancy between experiments and simulations. Meanwhile it is also worth examining present cavitation models, in order to obtain a more suitable one.

Acknowledgments

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Figure 7. Pressure and vapor distribution on the incipience of working condition A.

Figure 8. Vapor distribution on the critical of working condition A.
Figure 9. Pressure and vapor distribution on the incipience of working condition B.

Figure 10. Pressure and vapor distribution on the critical of working condition B.
References


