Modified convolution method to reconstruct particle hologram with an elliptical Gaussian beam illumination

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Abstract: Application of the modified convolution method to reconstruct digital inline holography of particle illuminated by an elliptical Gaussian beam is investigated. Based on the analysis on the formation of particle hologram using the Collins formula, the convolution method is modified to compensate the astigmatism by adding two scaling factors. Both simulated and experimental holograms of transparent droplets and opaque particles are used to test the algorithm, and the reconstructed images are compared with that using FRFT reconstruction. Results show that the modified convolution method can accurately reconstruct the particle image. This method has an advantage that the reconstructed images in different depth positions have the same size and resolution with the hologram. This work shows that digital inline holography has great potential in particle diagnostics in curvature containers.

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References and links
1. Introduction

Digital holography, as a real 3D imaging technique, can simultaneously provide the size [1, 2] and 3D position [1, 3] of the particle, and thereafter retrieve the 3D motion of the particle [4, 5]. Thus it has been widely applied to fluid and particle diagnostics [6], such as spray droplets [7], nonspherical particles [8, 9], bubbles [10], micro-particles [5, 11] or even microorganism [12, 13]. The interrogated objects are usually illuminated by the collimated plane wave or spherical wave to avoid aberration during the recording process, and then the object can be reconstructed by numerical diffraction of the hologram [1, 14]. In practice, the interrogated objects might be held in a complex container with curvature surface, and the collimated plane wave is transformed into a shaped beam after passing through the surface, such as elliptical Gaussian beam [15, 16]. This might cause aberration, such as the astigmatism in the optical system [15, 17–19]. Rather than the classical concentric rings, hologram fringes such as concentric elliptical rings, hyperbolic or even parallel lines have been observed in both simulation...
and experiments [15,16,20–23]. Consequently, the astigmatism distorts the recorded hologram, and hinder reconstructing the clear image with the classical method, such as Fresnel integral [1] and wavelet method [24].

The astigmatism should be eliminated in order to have a clear reconstructed image of the object. Various additional external lens systems were introduced between the curvature container and CCD to neutralize the window curvature effect during the recording of the hologram [17,18]. This suffers from the diversity of the practical applications and cannot be used in every experiment. The astigmatism can also be numerically compensated in the reconstruction process [14, 19–21, 25]. De Nicola [19] reconstructed a hologram, with a severe anamorphism caused by a reflection diffraction grating by modifying the propagation distance of the chirp kernel. The fractional Fourier transform (FRFT), which is suitable for linear chirp analysis, had been applied to reconstruct hologram of an opaque disk illuminated by an elliptical Gaussian beam by F. Nicolas [21]. The object was reconstructed through using the optional fractional orders. Then the FRFT was extended for wide angle reconstruction by Verrier [15, 16, 22] and for phase contrast holography by Marc Brunel [26]. But both the size and the resolution of the reconstructed image will vary with the optional fractional order due to the Fresnel diffraction, and it is inconvenient for post-processing, such as particle detecting and locating.

This work aims to develop a modified convolution reconstruction algorithm to reconstruct elliptical Gaussian beam illuminated particle hologram, with an advantage that the reconstructed images have the same size and resolution with the recorded hologram. Firstly, we will recall the formation and reconstruction of holograms of particles illuminated by elliptical Gaussian beam. And then both simulations hologram in framework of generalized Lorenz-Mie theory (GLMT) and experimental holograms of water droplets and non-spherical coal particle will be reconstructed with both the modified convolution method and FRFT for comparison. Transverse shift and magnification of the reconstructed particle image from the hologram with an elliptical Gaussian beam illumination will be also discussed.

2. Hologram formation

The interaction between a homogeneous spherical particle and an elliptical Gaussian beam can be rigorously described by GLMT [29]. If the beam radius is much larger than the particle size, the scattering light of an opaque spherical particle incident by an elliptical Gaussian beam is about the same as that by a plane wave [29]. In the Gabor in-line configuration, as shown in Fig. 1, the forward scattering of the particle is mostly the diffracted light, and it serves as the object wave and interferes with the undisturbed reference wave to form the hologram at the CCD plane. Thus the formation of the hologram can be approximately characterized by the scalar theory of diffraction. According to the Collins formula [30], the optical field at the hologram plane \((u,v)\) is

\[
U(u, v) = \iiint G_1(x, y)[1 - T(x, y)] \exp\left[\frac{i\pi}{\lambda B_x} (A_x u^2 - 2ux + D_x x^2)\right] \\
\exp\left[\frac{i\pi}{\lambda B_y} (A_y v^2 - 2vy + D_y y^2)\right] \, dx \, dy.
\]  

(1)

where \(G_1(x, y) = \exp\left(-\frac{x^2}{\omega_0^2} - \frac{y^2}{\omega_0^2}\right)\exp\left[-i\frac{\pi}{\lambda} \left(\frac{x^2}{R_{1x}^2} + \frac{y^2}{R_{1y}^2}\right)\right]\) is the laser beam incident on the particle, with \(\omega_{0x,0y}, R_{1x,1y}\), being the beam radius and wave curvature in the \(x\) and \(y\) directions respectively. \(A_{x,y}, B_{x,y}, D_{x,y}\) are the elements of the ABCD transform matrix \(\begin{pmatrix} A_{x,y} & B_{x,y} \\ C_{x,y} & D_{x,y} \end{pmatrix}\) from the particle to the CCD in the \(x\) and \(y\) directions. \(T(x, y) = \sum_{k=1}^{10} A_k \exp\left[-B_k (x^2 + y^2)/r^2\right]\) denotes the particle expressed as a complex Gaussian expansion [31]. It can provide analytical
expressions of the hologram with clear physical meaning. This integral can be divided into two parts, the reference wave and the object wave. The reference wave \( R \) is

\[
R(u,v) = \iint G_1(x,y) \exp[i \frac{\pi}{\lambda B_x} (A_x u^2 - 2ux + D_x x^2)] \exp[i \frac{\pi}{\lambda B_y} (A_y v^2 - 2vy + D_y y^2)] dxdy
\]

\[= C_{ref} \exp\left(\frac{i\pi D_x x^2}{\lambda B_x} + \frac{i\pi D_y y^2}{\lambda B_y}\right) \exp\left[\frac{\pi x^2}{\lambda B_x (-\frac{\lambda B_y}{\pi \omega y_1} + \frac{\mu y}{\pi r_1} - iA_x)} + \frac{\pi y^2}{\lambda B_y (-\frac{\lambda B_y}{\pi \omega y_1} + \frac{\mu y}{\pi r_1} - iA_y)}\right] \quad (2)
\]

where the constant \( C_{ref} = \pi / \sqrt{(-\frac{\lambda B_x}{\pi \omega x_1} + \frac{i\mu x}{\pi r_1} - iA_x)(-\frac{\lambda B_y}{\pi \omega y_1} + \frac{i\mu y}{\pi r_1} - iA_y)} \). The reference wave \( R \) is still an elliptical Gaussian beam, and the objective wave \( O \) is

\[
O(u,v) = \iint G_1(x,y) T(x,y) \exp[i \frac{\pi}{\lambda B_x} (A_x u^2 - 2ux + D_x x^2)] \exp[i \frac{\pi}{\lambda B_y} (A_y v^2 - 2vy + D_y y^2)] dxdy
\]

\[= C_{obj} \exp\left(\frac{i\pi D_x x^2}{\lambda B_x} + \frac{i\pi D_y y^2}{\lambda B_y}\right) \sum_{k=1}^{10} A_k \exp\left[\frac{\pi x^2}{\lambda B_x (-\frac{\lambda B_y}{\pi \omega y_1} + \frac{\mu y}{\pi r_1} - iA_x)} + \frac{\pi y^2}{\lambda B_y (-\frac{\lambda B_y}{\pi \omega y_1} + \frac{\mu y}{\pi r_1} - iA_y)}\right] \quad (3)
\]

where the constant \( C_{obj} = \pi \).

Fig. 1. Schematics of digital particle holography with elliptical Gaussian beam illumination.

The light intensity in the hologram recorded by the CCD is

\[
I_{holo}(u,v) = U(u,v) \cdot \overline{U}(u,v) = R \cdot \overline{R} + O \cdot \overline{O} + R \cdot \overline{O} + O \cdot \overline{R}
\]

(4)

The \( R \cdot \overline{R} \) and \( O \cdot \overline{O} \) are the directly transmitted light and cannot be used to reconstruct the object. The \( O \cdot \overline{R} \) and \( R \cdot \overline{O} \) determine the interference pattern and can be used to reconstruct the virtual and real image of the object respectively.

3. Modified convolution reconstruction

The frequency of the interference pattern is determined by the phase of the \( O \cdot \overline{R} \) and \( R \cdot \overline{O} \).
account. For an elliptical Gaussian beam, the optical field using the classical methods should be rescaled to take the magnifications into account. For a plane wave propagating through the flat surface, the hologram, the linear chirp should be removed by conjugating the reference wave. From Eq. 5, the $z_{(x,y),eq} = B_{(x,y)} \cdot R_{(x,y)}$ can be treated as the equivalent propagation distance from the particle to the CCD, which determines the frequency of the fringes in $x$ and $y$ directions in the hologram. For a plane wave propagating through the flat surface, $\omega_{x1,y1} \to \infty$, $R_{x1,y1} \to \infty$, the $R_{x,y} = 1$, and $z_{(x,y),eq}$ equal to the wave propagation distance $z$. The hologram can be reconstructed with the classical methods. For a circular Gaussian beam, $R_x$ equals to $R_y$ but not 1, the $z_{x,eq} = z_{y,eq}$ with the same magnifications in both $x$ and $y$ directions. The fringe patterns in the hologram of a spherical particle are still composed of concentric rings. The reconstructed object field using the classical methods should be rescaled to take the magnifications into account. For an elliptical Gaussian beam, the $R_x \neq R_y$, and the $B_x \neq B_y$, it can be interpreted that the beam has propagated different equivalent distances from the particle to the CCD. This results in the different magnifications and the spatial frequency of the fringes in the transverse directions. Thus, the hologram cannot be simultaneously focused in both $x$ and $y$ directions at a certain depth position with the classical methods for reconstruction. Figure 2 shows a simulated hologram of a spherical particle illuminated by an elliptical Gaussian beam, and the image slices reconstructed using the classical convolution method. The object is focused at $z = 7.2$ cm in the $x$ direction (in Fig. 2(b)) and at $z = 4.0$ cm in the $y$ direction (in Fig. 2(c)). However, no clear particle image can be observed in the two reconstructed images.

\[
\arg(O \cdot R + R \cdot O) = \Im\left[\frac{\pi x^2}{\lambda \omega_x \omega_y} - \frac{i \omega_x}{\lambda \omega_x \omega_y} + \frac{i A_x}{\lambda} + \frac{i \omega_y}{\lambda} - \frac{i A_y}{\lambda}\right] = -\frac{\pi x^2}{\lambda \omega_x \omega_y} - \frac{\pi y^2}{\lambda \omega_x \omega_y} = \pi x^2 Rx + \pi y^2 Ry = \pi x^2 Rx + \pi y^2 Ry = \pi x^2 + \pi y^2 (5)
\]

where $\Im$ denotes the imaginary part of a complex number. To reconstruct the object from the hologram, the linear chirp should be removed by conjugating the reference wave. From Eq. 5, the $z_{(x,y),eq} = B_{(x,y)} \cdot R_{(x,y)}$ can be treated as the equivalent propagation distance from the particle to the CCD, which determines the frequency of the fringes in $x$ and $y$ directions in the hologram. For a plane wave propagating through the flat surface, $\omega_{x1,y1} \to \infty$, $R_{x1,y1} \to \infty$, the $R_{x,y} = 1$, and $z_{(x,y),eq}$ equal to the wave propagation distance $z$. The hologram can be reconstructed with the classical methods. For a circular Gaussian beam, $R_x$ equals to $R_y$ but not 1, the $z_{x,eq} = z_{y,eq}$ with the same magnifications in both $x$ and $y$ directions. The fringe patterns in the hologram of a spherical particle are still composed of concentric rings. The reconstructed object field using the classical methods should be rescaled to take the magnifications into account. For an elliptical Gaussian beam, the $R_x \neq R_y$, and the $B_x \neq B_y$, it can be interpreted that the beam has propagated different equivalent distances from the particle to the CCD. This results in the different magnifications and the spatial frequency of the fringes in the transverse directions. Thus, the hologram cannot be simultaneously focused in both $x$ and $y$ directions at a certain depth position with the classical methods for reconstruction. Figure 2 shows a simulated hologram of a spherical particle illuminated by an elliptical Gaussian beam, and the image slices reconstructed using the classical convolution method. The object is focused at $z = 7.2$ cm in the $x$ direction (in Fig. 2(b)) and at $z = 4.0$ cm in the $y$ direction (in Fig. 2(c)). However, no clear particle image can be observed in the two reconstructed images.

Fig. 2. Hologram of a particle with elliptical Gaussian beam illumination and its reconstructed images with the classical convolution method. (a. simulated hologram, b. reconstructed image at $z = 7.2$ cm, c. reconstructed image at $z = 4.0$ cm.)

To make the object focused at the same reconstructed plane at a certain depth, we modify the kernel of the convolution method as follows

\[
g(x,y,u,v) = \frac{i \exp(-ik\sqrt{S_x^2(u-x)^2 + S_y^2(y-v)^2 + z^2})}{\lambda \sqrt{S_x^2(u-x)^2 + S_y^2(y-v)^2 + z^2}} (6)
\]

where $k = 2\pi/\lambda$, and the scale parameters $S_x$, $S_y$ are related with the transfer matrix (equivalent recording distance).
\[ S_x^2 = \frac{z}{B_x R_x}, \quad S_y^2 = \frac{z}{B_y R_y}. \]  

(7)

Note that \( S_x^2 \) and \( S_y^2 \) could be positive or negative. In this work the light propagates from the particle to the CCD in the free space, so the \( B_{xy} = z \). The object can be reconstructed by a convolution of the modified kernel and the hologram. According to the convolution theorem, this process can be speeded using the fast Fourier transform (FFT).

\[ I_r(x, y) = I_{\text{holo}}(x, y) \otimes g(x, y, u, v) = F^{-1}[F(I_{\text{holo}}) \cdot F(g)] \]

(8)

where \( \otimes \) is the convolution operation, and \( F \) and \( F^{-1} \) denote the forward and inverse Fourier transform respectively. The reconstructed image has the same size and resolution of the recorded hologram. Note that the physical resolution \( \delta_{xy} \) of the reconstructed image is determined by diffraction [27], \( \delta_{xy} = \lambda z / N \delta \), with \( N \) the sampling points number and \( \delta \) the sampling pitch of the recorded hologram.

From the angular point of view, the transfer function \( G_z(u, v) = F(g) \) can be obtained by direct calculation of the analytic expansion [28]:

\[ G_z(u, v) = \begin{cases} \exp(-i2\pi z \sqrt{\frac{1}{\lambda^2 x^2} - \frac{u^2}{\delta_{xy}^2} - \frac{v^2}{\delta_{xy}^2}}), & \frac{1}{\lambda^2 x^2} > \frac{u^2}{\delta_{xy}^2} + \frac{v^2}{\delta_{xy}^2} \\ 0, & \text{otherwise} \end{cases} \]

(9)

Then the hologram can be reconstructed by \( I_r(x, y) = F^{-1}[F(I_{\text{holo}}) \cdot G] \), saving one Fourier transform.

The scaling parameters \( S_{xy} \) and the FRFT orders \( \alpha_{xy} \) are related to each other. The optimal fractional orders to reconstruct the object proposed by the holography group in CORIA [16, 20, 21] are:

\[ \alpha_{xy} = \frac{2}{\pi} \arctan \left( \frac{\lambda B_{xy}}{N_{xy} \delta_{xy}^2 (M_{xy} - D_{xy})} \right) \]

(10)

where \( N_{xy} \) the sampling points number and \( \delta_{xy} \) the sampling pitches in the \( x \) and \( y \) directions. The \((M_{xy} - D_{xy})\) is the scaling coefficients of the quadratic phase, Comparing Eq. (2, 5) and Eq. (B8, B9) [16], we can obtain \( \frac{1}{R_{xy}} = (M_{xy} - D_{xy}) \) with uncomplicated formula derivation. Thus, the relation between \( S_{xy} \) and \( \alpha_{xy} \) can be obtained as follows:

\[ S_{xy}^2 \tan \frac{\pi \alpha_{xy}}{2} = \frac{\lambda z(x, y)_{eq}}{N_{xy} \delta_{xy}^2} = \text{constant} \]

(11)

4. Simulated holograms

The modified convolution method was firstly validated by the simulated holograms. Three typical fringe patterns (concentric elliptical rings, hyperbolic lines, and parallel lines) have been observed in the hologram of particles illuminated by the elliptical Gaussian beam. The simulated hologram of homogeneous, spherical particles was computed in the framework of the GLMT [23, 29]. Figure 3(a) shows a simulated hologram of an opaque particle illuminated by an elliptical Gaussian beam with the wave radius of the beam waist of \( \alpha_0 = 2 \mu \text{m} \) and \( \alpha_{0y} = 3 \) mm in the \( x \) and \( y \) directions respectively. The particle with a diameter of \( 140 \mu \text{m} \) located at \( z = 5.0 \) cm. The wave radii of the beam at the depth position of the particle were \( 4.2 \) mm and \( 3.0 \) mm, with the wave curvature of \( 5 \) cm and \( 5.65 \times 10^4 \) m in the \( x \) and \( y \) directions respectively. Detailed parameters of the simulated hologram are given in table 1. The hologram is characterized with elliptical fringes. The hologram was reconstructed through using the modified convolution method, with the \( S_x^2 = 0.556, S_y^2 = 1 \). A focused particle image was obtained.
at $z = 4.0$ cm, as shown in Fig. 3(b). The hologram was also reconstructed by using fractional Fourier transform (FRFT) for comparison. The particle was focused at optional fractional orders $\alpha_x = 0.743$, $\alpha_y = 0.582$ in FRFT. The scaling factors $S_x$, $S_y$ and optimal fractional orders $\alpha_x$, $\alpha_y$ satisfy the relation in Eq. (11). By rescaling the reconstructed image to make its resolution equaling to the recorded hologram, Fig. 3(c) shows the selected ROI region of the reconstructed image. Comparisons between the reconstructed images in Fig. 3(b) and Fig. 3(c) show that they are the same.

![Fig. 3. Particle hologram and its reconstructed image using modified convolution and FRFT. (a. simulated hologram, b. convolution reconstruction with $S_x = 0.556$, $S_y = 1$, c. FRFT reconstruction with optimal fractional orders $\alpha_x = 0.743$, $\alpha_y = 0.582$.)](image_url)

Figure 4(a) shows a simulated hologram with the particle located at $z = -2.3$ cm, and the detailed simulation parameters are listed in Table 1. Hyperbolic fringes can be observed in the hologram. The in-focus image of the particle was reconstructed at $z = 7.3$ cm using the modified convolution method with $S_x = -0.460$, $S_y = 1$, and at optional fractional orders $\alpha_x = -0.878$, $\alpha_y = 0.746$ using fractional Fourier transform, as shown in Fig. 4(b) and Fig. 4(c) respectively.

![Fig. 4. Particle hologram and its reconstructed image using modified convolution and FRFT. (a. simulated hologram, b. convolution reconstruction with $S_x = -0.460$, $S_y = 1$, c. FRFT reconstruction with optimal fractional orders $\alpha_x = -0.878$, $\alpha_y = 0.746$.)](image_url)

Simulated holograms of transparent spherical particles illuminated by an elliptical Gaussian beam are also used to test the modified convolution method. The imaginary part of the refractive index of the droplet is zero, with details of the simulations given in Table 1. The reconstructed objects can be focalized by both modified convolution and FRFT reconstruction. It is worth a mention that there is a bright point in the center of the reconstructed dark image of the particle, as shown in Fig. 5 and Fig. 6. This might be caused by the light focusing effect of the transparent droplet.

The above results demonstrate that the modified convolution method can reconstruct the in-focus particle images from the holograms of both opaque and transparent particles illuminated...
Fig. 5. Hologram of droplet and its reconstructed image using modified convolution and FRFT. (a. simulated hologram, b. convolution reconstruction with $S_2^x = 0.540$, $S_2^y = 1$, c. FRFT reconstruction with optimal fractional orders $\alpha_x = 0.601$, $\alpha_y = 0.408$.)

Fig. 6. Hologram of droplet and its reconstructed image using modified convolution and FRFT. (a. simulated hologram, b. convolution reconstruction with $S_2^x = -0.540$, $S_2^y = 1$, c. FRFT reconstruction with optimal fractional orders $\alpha_x = -0.865$, $\alpha_y = 0.758$.)

Table 1. Detailed parameters of simulated holograms, $\omega_{0x}=2\mu m$, $\omega_{0y}=3\times10^{-3}mm$ in $x$ and $y$ directions, CCD pixel=4\mu m. (particle diameter: $d / \mu m$, refractive index: $n$, CCD position: $z_{CCD} / cm$, wave curvature: $R_1 / cm$ and $R_2 / m$, beam radius at the particle: $\omega_{1x}$ /mm and $\omega_{1y}$ /mm, beam radius at the CCD: $\omega_{x,CCD} / mm$ and $\omega_{y,CCD} / mm$, reconstructed particle diameter: $d_x$ and $d_y / \mu m$.)

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by an elliptical Gaussian beam. Figure 7(a) shows the simulated hologram of a cloud of particles, and the comparisons between the simulated and reconstructed particles are shown in Fig. 7(b). The reconstructed particles located at the same depth positions as the simulated particles. However, the reconstructed particle image might be different from the actually recorded particle in both morphology and transversal positions due to astigmatism. Since the beam with the same wavelength and curvature is used for reconstruction, no transverse shift is introduced during the reconstruction. The magnification in particle size and transverse shift in position are caused during the propagation of the elliptical Gaussian beam propagating from the particle to the CCD [20, 22]. The magnification can be evaluated from the beam waist radius ratio at the particle and CCD plane:

\[ M_{\text{mag},(x,y)} = \frac{\omega_{(x,y),\text{CCD}}}{\omega_{1,y1}} \]  

(12)

Indeed, this transverse shift and magnification might even be different in the \(x\) and \(y\) directions. For a spherical particle, the cross-section of the reconstructed in-focus image might be elliptical, as shown in Fig. 2-7. Table 1 compares the ratio of simulated particle size with its reconstructed particle size, and results agree well with Eq. (12). In the far field, both the beam radius and wave curvature increase linearly with the distance, and the wave can be approximately considered as a spherical wave centered at the beam waist. The transverse shift can be evaluated according to geometric optics [22]. Note that if the particle is located after the beam waist and the CCD before the beam waist, the reconstructed real image of the particle from the hyperbolic hologram fringes is upside down and flipped left to right, as the particles shown in Fig. 7.

![Fig. 7. Effect of magnification and position shift. (a. hologram of a cloud of particles with elliptical Gaussian beam illumination, b. comparison of the 3D position and cross-section between the simulated and reconstructed particles.)](image)

5. Experiments

Experimental holograms of transparent water droplets and nonspherical opaque coal particles were captured with the typical Gabor inline configuration, as shown in Fig. 1. A circular Gaussian beam, with the beam radius of 5 mm, was transformed into an elliptical Gaussian beam by a cylinder lens, which has a focus of 100 mm and infinite in the \(x\) and \(y\) direction respectively. Holograms of particles were recorded by the CCD (LaVision ImagePro) with a squared pixel
size of 7.4 μm. Two particle fields were used: one was the sprayed water droplets, and the other was coal particles.

![Fig. 8. Experimental holograms with typical elliptical fringes, hyperbolic fringes and parallel line fringes. (Water droplet located before (a) and after (b) the beam waist, Coal particle located before (c) and after (d) the beam waist.)](image)

Figure 8 shows the obtained holograms with water droplets and coal particles located at different depth positions with elliptical Gaussian beam illumination. The concentric elliptical rings and hyperbolic fringes were observed in the holograms for both water droplets (in Fig. 8(a-b)) and coal particles (in Fig. 8(c-d)), and this three typical fringe patterns correspond to the particle locating before, after and at (or very near to) the beam waist respectively. The fringes were not rigorously elliptical in Fig. 8(c) or hyperbolic in Fig. 8(d) due to the nonsphericity of the coal particles. The holograms in Fig. 8(a-b) and Fig. 8(c-d) were reconstructed through using the modified convolution method and FRFT, as shown in Fig. 9. Comparisons show that the in-focus image can be reconstructed by both methods at the same depth position.

6. Parallel fringes

For a circular Gaussian beam passing through a cylindrical lens, as shown in Fig. 1, the elliptical Gaussian beam highly is focused in the horizontal (x) direction, while not focused in the vertical (y) direction. Supposing a particle locating at or very near to the beam waist of the elliptical Gaussian beam, From Eq. (5), in the x direction, the $\omega_{x1}$ and the particle diameter $d$ are about at the same size order of magnitude, and the radius of curvature $R_{x1} \rightarrow \infty$. Thus the $R_x$ is much larger than 1 (up to $10^3$ or larger), and the equivalent propagation distance $z_{(x),eq}$ far greater than the actual propagation distance $z$. In the y direction, while $\omega_{y1} \gg d$, and the radius of curvature $R_{y1} \rightarrow \infty$. Thus the $R_y \simeq 1$, and $z_{(y),eq} = z$. From the quadratic phase term in Eq. (5), the frequency of the fringes in the x direction is not only much smaller than that in the y direction, but also so small that the CCD could not even sample a period. Therefore, the recorded hologram fringes of a particle are composed of parallel fringes. Figure 10(a) and 10(b) shows the simulated and experimental holograms with parallel fringes.

Since only the parallel fringes conveys information of the particle and the equivalent propagation distance of the parallel fringes equals to the actual distance from the particle to the CCD, the particle image should be reconstructed with the classical convolution method with the scaling parameters $S_{x,y} = 1$, as shown in Fig. 10(c) and 10(d). The focused particle image

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Fig. 9. Reconstructed image holograms in Fig. 8 using modified convolution and fractional Fourier transform (a. modified convolution reconstruction of Fig. 8(a) with $S_x^2 = 0.625$, $S_y^2 = 1$, $z=51.85\,\text{mm}$, b. FRFT reconstruction of Fig. 8(a) with optimal fractional orders $\alpha_x = 0.478$, $\alpha_y = 0.336$, c. modified convolution reconstruction of Fig. 8(b) with $S_x^2 = -0.693$, $S_y^2 = 1$, $z=231.2\,\text{mm}$, d. FRFT reconstruction of Fig. 8(b) with optimal fractional orders $\alpha_x = -0.834$, $\alpha_y = 0.736$, e. modified convolution reconstruction of Fig. 8(c) with $S_x^2 = 0.392$, $S_y^2 = 1$, $z=96.6\,\text{mm}$, f. FRFT reconstruction of Fig. 8(c) with optimal fractional orders $\alpha_x = -0.785$, $\alpha_y = 0.535$, h. modified convolution reconstruction of Fig. 8(d) with $S_x^2 = -0.324$, $S_y^2 = 1$, $z=206\,\text{mm}$, i. FRFT reconstruction of Fig. 8(d) with optimal fractional orders $\alpha_x = -0.915$, $\alpha_y = 0.750$)

Fig. 10. Holograms with parallel fringes. (a. simulated hologram with parallel line fringes, with $\omega_x = 16\,\mu\text{m}$, $\omega_y = 3\,\text{mm}$, particle diameter $d = 10\,\mu\text{m}$, particle located at the beam waist center, CCD position $z_{\text{CCD}} = 5.0\,\text{cm}$, b. experimental hologram of water droplet with parallel line fringes, c. reconstructed image of Fig. 10(a) at $z = 5.0\,\text{cm}$, d. reconstructed image of Fig. 10(b) at $z = 134.0\,\text{mm}$.)
is distorted to a line, due to the huge magnification in the $x$ direction.

7. Conclusion

A modified convolution method is proposed to compensate the astigmatism and applied to reconstructed particle hologram with an elliptical Gaussian beam illumination. The scale parameters $S_x$ and $S_y$ are introduced to compensate the different spatial frequencies of the fringe patterns in the $x$ and $y$ directions caused by the astigmatism. This algorithm is validated by both simulated and experimental holograms of transparent droplets and opaque particles. The reconstructed images with the modified convolution method are compared with those using FRFT reconstruction, and results show that the modified convolution method can accurately reconstruct the particle image. Transverse shift and magnification of the hologram by an elliptical Gaussian beam are also discussed. This method has the advantage that both the size and resolution of the reconstructed image equal to the recorded hologram, and thus the reconstructed images can be easily extended for post-processing.

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