Effects of hadronic potentials on elliptic flows in relativistic heavy ion collisions

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Within the framework of a multiphase transport (AMPT) model that includes both initial partonic and final hadronic interactions, we show that including mean-field potentials in the hadronic phase leads to a splitting of the elliptic flows of particles and their antiparticles, thus providing a plausible explanation of the different elliptic flows between p and $\bar{p}$, $K^+$ and $K^-$, and $\pi^+$ and $\pi^-$ observed in the recent Beam Energy Scan (BES) program at the Relativistic Heavy-Ion Collider (RHIC).

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Understanding the phase diagram of strongly interacting matter is one of the main goals of the experiments that are being carried out in heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) [1]. For collisions at its highest center-of-mass energy of $\sqrt{s_{NN}} = 200$ GeV, convincing evidence has been established that a strongly interacting quark-gluon plasma (sQGP) is formed in these collisions [2]. The baryon chemical potential of the formed sQGP is small, and according to calculations based on lattice quantum chromodynamics (QCD), the phase transition between the sQGP and hadronic matter for such a small baryon chemical potential is a smooth crossover transition [3]. Theoretical studies based on various models have shown, on the other hand, that this crossover transition would change to a first-order phase transition when the baryon chemical potential is increased [1]. To search for the critical point in the baryon chemical potential and temperature plane at which such a transition occurs, the Beam Energy Scan (BES) program involving Au + Au collisions at lower energies of $\sqrt{s_{NN}} = 7.7, 11.5$, and $39$ GeV has recently been carried out. Although no definitive conclusions on the location of the critical point in the QCD phase diagram have been obtained from these experiments, several interesting phenomena have been observed [4]. Among them are the smaller elliptic flows of $\bar{p}$, $K^-$, and $\pi^+$ than those of $p$, $K^+$, and $\pi^-$, respectively. The differences decrease with increasing collision energy and become essentially very small at higher collision energies [5]. These surprising results were recently attributed to the different elliptic flows of transported and produced partons during the initial stage of heavy ion collisions [6]. Also, it was suggested that the chiral magnetic effect induced by the strong magnetic field in noncentral collisions could be responsible for the observed difference between the elliptic flows of $\pi^+$ and $\pi^-$ [7].

It is known from heavy ion collisions at lower collision energies at SIS/GSI and AGS/BNL that the elliptic flow of nucleons is affected not only by their scattering but also by their mean-field potentials in hadronic matter [8]. In particular, in collisions at a few AGeV, when participating nucleons are not blocked by spectator nucleons, an attractive (repulsive) potential is found to result in a smaller (larger) elliptic flow. This is because particles with attractive potentials are more likely to be trapped in the system and move in the direction perpendicular to the participant plane, while those with repulsive potentials are more likely to leave the system and move along the participant plane, thus reducing and enhancing their respective elliptic flows. Also, the potentials of a particle and its antiparticle are different, and they generally have opposite signs at high densities [9,10]. As a result, particles and antiparticles are expected to have different elliptic flows in heavy ion collisions when the produced matter has a nonzero baryon chemical potential. Furthermore, the difference between the potentials of a particle and its antiparticle diminishes with decreasing baryon chemical potential, so their elliptic flows are expected to become similar in higher energy collisions when more antiparticles are produced. These effects are all consistent with what were seen in the experimental data from the BES program.

In the present paper, we study quantitatively the elliptic flows of particles and their antiparticles at BES energies by extending a multiphase transport (AMPT) model [11] to include the potentials of baryons, kaons, and pions as well as their antiparticles. The AMPT model is a hybrid model with initial particle distributions generated by the heavy-ion jet interaction generator (HIJING) model [12] via the Lund string fragmentation model. In the string melting version of the AMPT model, which is used in the present study, hadrons produced from excited strings in the HIJING model are converted to their constituent or valence quarks and antiquarks, and their evolution in time and space is then modeled by Zhang’s parton cascade (ZPC) model [13]. Differing from previous applications of the AMPT model for heavy ion collisions at higher energies, the parton scattering cross section and the ending time of the partonic stage are adjusted in the present study to approximately reproduce measured elliptic flows and the hadronic energy density ($\sim 0.30–0.35$ GeV/fm$^3$) at the extracted baryon chemical potential and temperature.
at chemical freeze-out [14]. Specifically, we take the parton scattering cross sections to be isotropic, with the values 3, 6, and 10 mb, and the ending times of the partonic stage to be 3.5, 2.6, and 2.9 fm/c for collisions at \( \sqrt{s_{NN}} = 7.7, 11.5, \) and 39 GeV, respectively. At hadronization, quarks and antiquarks in the AMPT model are converted to hadrons via a spatial coalescence model, and the scatterings between hadrons in the hadronic stage are described by a relativistic transport (ART) model [15] that has been extended to also include particle-antiparticle annihilations and their reverse reactions.

We take the nucleon and antinucleon potentials from the relativistic mean-field model used in the relativistic Vlasov-Uehling-Uhlenbeck transport model [16], that is,

\[
U_{N,\bar{N}}(\rho_B, \rho_{\bar{B}}) = \Sigma_0(\rho_B, \rho_{\bar{B}}) \pm \Sigma_1(\rho_B, \rho_{\bar{B}}),
\]

in terms of the nucleon scalar \( \Sigma_0(\rho_B, \rho_{\bar{B}}) \) and vector \( \Sigma_1(\rho_B, \rho_{\bar{B}}) \) self-energies in hadronic matter of baryon density \( \rho_B \) and antibaryon density \( \rho_{\bar{B}} \). The “+” and “−” signs are for nucleons and antinucleons, respectively. We note that nucleons and antinucleons contribute both positively to \( \rho_B \) and \( \rho_{\bar{B}} \), respectively.

\[
\rho = \frac{m_{\pi}}{2} (aK + bK\rho_B, \rho_{\bar{B}})
\]

and \( \omega_0 = \sqrt{m_K^2 + p^2} \), where \( m_K \) is the kaon mass and \( aK = 0.22 \text{ GeV}^2\text{fm}^3 \), \( aK = 0.45 \text{ GeV}^2\text{fm}^3 \), and \( bK = 0.33 \text{ GeV}^2\text{fm}^3 \) are empirical parameters taken from Ref. [17]. In the above, \( \rho_s \) is the scalar density, which can be determined from \( \rho_B \) and \( \rho_{\bar{B}} \) through the effective interaction used for describing the properties of nuclear matter, and \( \rho_B^{\text{net}} = \rho_B - \rho_{\bar{B}} \) is the net baryon density. The “+” and “−” signs are for kaons and antikaons, respectively.

The pion potentials are related to their self-energies \( \Pi_{\pi}^{\pm \pm} \) according to \( U_{\pi,\bar{\pi}} = \Pi_{\pi}^{\pm \pm} / (2m_\pi) \), where \( m_\pi \) is the pion mass. In Ref. [18], the contribution of the pion-nucleon s-wave interaction to the pion self-energy has been calculated up to the two-loop order in chiral perturbation theory. In asymmetric nuclear matter of proton density \( \rho_p \) and neutron density \( \rho_n \), the resulting \( \pi^- \) and \( \pi^+ \) self-energies are given, respectively, by

\[
\Pi^-_{\pi}(\rho_p, \rho_n) = \rho_n[T^+_{\pi N} - T^+_{\bar{\pi} N}] - \rho_p[T^-_{\pi N} + T^-_{\bar{\pi} N}]
\]

\[
+ \Pi_{\text{cor}}(\rho_p, \rho_n) + \Pi_{\text{iso}}(\rho_p, \rho_n).
\]

\[
\Pi^+_{\pi}(\rho_p, \rho_n) = \Pi^-_{\pi}(\rho_n, \rho_p).
\]

In the above, \( T^\pm \) are the isospin-even and isospin-odd \( \pi N \) s-wave scattering \( T \) matrices, which are given by the one-loop contribution in chiral perturbation theory and have the empirical values \( T^+_{\pi N} \approx 1.847 \text{ fm} \) and \( T^-_{\pi N} \approx -0.045 \text{ fm} \), extracted from the energy shift and width of the 1s level in a pionic hydrogen atom; \( \Pi_{\text{cor}} \) is due to the relativistic correction; and \( \Pi_{\text{iso}} \) is the contribution from the two-loop order in chiral perturbation theory. Their expressions can be found in Ref. [18]. For nucleon resonances and strange baryons in hadronic matter, we simply extend the above result by treating them as neutron- or proton-like according to their isospin structure [15] and light-quark numbers.

In Ref. [18], the contribution of the pion-nucleon p-wave interaction [19], which is expected to reduce and enhance, respectively, the \( \pi^- \) and \( \pi^+ \) potential difference due to the s-wave interaction in neutron-rich and proton-rich matter [20], since its inclusion in the transport model is highly nontrivial [21].

In Fig. 1, we show the \( N \), \( K \), and \( \bar{K} \) potentials as functions of \( \rho_B \) and \( \rho_{\bar{B}} \), and the \( \pi^- \) potential as a function of neutron-like and proton-like densities \( \rho_n \) and \( \rho_p \). The \( \bar{N} \) and \( \pi^+ \) potentials are related to those of \( N \) and \( \pi^- \) by \( U_{\bar{N}}(\rho_B, \rho_{\bar{B}}) = U_N(\rho_B, \rho_{\bar{B}}) \) and \( U_{\pi^+}(\rho_p, \rho_n) = U_{\pi^-}(\rho_n, \rho_p) \). In the absence of antibaryons, the \( N \) potential is slightly attractive, while that of \( \bar{N} \) is strongly attractive, with values of about -60 and -260 MeV, respectively, at normal nuclear matter density \( \rho_0 = 0.16 \text{ fm}^{-3} \). The latter is similar to that determined from the nonlinear derivative model for small antinucleon kinetic energies [22], and is also consistent with those values extrapolated from experimental data [23–28]. For pions in neutron-rich nuclear matter, the potential is weakly repulsive and attractive for \( \pi^- \) and \( \pi^+ \), respectively, and the strength at \( \rho_0 \) and isospin asymmetry \( \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = 0.2 \) is about 14 MeV for \( \pi^- \) and -1 MeV for \( \pi^+ \). In antibaryon-free matter, the \( K \) potential is slightly repulsive while the \( \bar{K} \) potential is deeply attractive, and their values at \( \rho_0 \) are about 20 and -120 MeV, respectively, similar to those extracted.
FIG. 2. (Color online) Time evolution of the baryon density (a), antibaryon density (b), neutron-like density (c), and isospin asymmetry (d) in the central region of the hadronic phase from the AMPT model for Au + Au collisions at $b = 8$ fm and $\sqrt{s_{NN}} = 7.7$, 11.5, and 39 GeV.

from the experimental data [29–32] and used in the previous study [33].

Figure 2 displays the time evolution of the baryon density (a), antibaryon density (b), neutron-like density (c), and isospin asymmetry (d) in the central region of the hadronic phase from the AMPT model for Au + Au collisions at impact parameter $b = 8$ fm for the three different BES energies of $\sqrt{s_{NN}} = 7.7$, 11.5, and 39 GeV. It is seen that the baryon density decreases while the antibaryon density increases with increasing collision energy, resulting in a decrease of the net baryon density with increasing collision energy. Also, the isospin asymmetry is very small in the hadronic phase for all three energies due to the considerable number of $\Lambda$ hyperons, which do not carry isospin, and the larger number of $\pi^-$ than $\pi^+$ produced in the collisions.

The differential elliptic flows of $p$, $K^+$, and $\pi^+$ as well as their antiparticles with and without hadronic potentials at three different BES energies from the string melting AMPT model are shown in Fig. 3. They are calculated with respect to the participant plane, that is $v_2 = \langle \cos[2(\phi - \Psi_2)] \rangle$, where $\phi = \tan^{-1}(p_y, p_x)$ is the azimuthal angle, $\Psi_2 = \tan^{-1}(r_p^2 \sin 2\phi_p, (r_p^2 \cos 2\phi_p) + \pi)/2$ is the angle of the participant plane, and $r_p$ and $\phi_p$ are the polar coordinates of the participants. Without hadronic potentials the elliptic flows from the AMPT model are similar for particles and their antiparticles. Including hadronic potentials increases slightly the $p$ and $\bar{p}$ elliptic flows at $p_T < 0.5$ GeV/c, while it reduces slightly (strongly) the $p$ ($\bar{p}$) elliptic flow at higher $p_T$, consistent with the expectations from the relative strength of the attractive potentials for $N$ and $\bar{N}$ shown in Fig. 1. Hadronic potentials also increase slightly the elliptic flow of $K^+$ while reducing mostly that of $K^-$, again consistent with what is expected from the $K$ and $\bar{K}$ potentials shown in Fig. 1. In addition, the effect from the potentials on the elliptic flow decreases with increasing collision energy, which is consistent with the decreasing baryon density and net baryon density of produced hadronic matter with increasing collision energy shown in Fig. 2. The differences between the differential elliptic flows of $p$ and $\bar{p}$, and between those of $K^+$ and $K^-$ below $\sqrt{s_{NN}} = 11.5$ GeV are qualitatively consistent with the experimental data [4], while that of $\pi^-$ and $\pi^+$ is small in all three energies due to the small isospin asymmetries shown in Fig. 2.

Our results for the relative $p_T$-integrated $v_2$ difference between particles and their antiparticles, defined by $[v_2(P) - v_2(\bar{P})]/v_2(P)$, with and without hadronic potentials are shown in Fig. 4. These differences are very small in the absence of hadronic potentials. Including hadronic potentials increases the relative $v_2$ difference between $p$ and $\bar{p}$ and between $K^+$ and $K^-$ up to about 30% at 7.7 GeV and 20% at 11.5 GeV, but negligibly at 39 GeV. These results are qualitatively consistent with the measured values of about 63% and 13% at 7.7 GeV, 44% and 3% at 11.5 GeV, and 12% and 1% for the relative $v_2$ difference between $p$ and $\bar{p}$ and between $K^+$ and $K^-$.
respectively [4]. Similar to the experimental data, the relative $v_2$ difference between $\pi^+$ and $\pi^-$ is negative at all energies after including their potentials, although ours have smaller magnitudes. We have also found that, as seen in experiments [4], the relative $v_2$ difference between $\Lambda$ hyperons and $\bar{\Lambda}$ is smaller than that between $p$ and $\bar{p}$, because the $\Lambda(\bar{\Lambda})$ potential is only 2/3 of the $p(\bar{p})$ potential.

To summarize, we have studied the elliptic flows of $p$, $K^+$, $\pi^+$ and their antiparticles in heavy ion collisions at BES energies by extending the string melting AMPT model to include their mean-field potentials in the hadronic stage. Because of the more attractive potentials of $\bar{p}$ than $p$, the attractive $K^-$ and repulsive $K^+$ potentials, and the slightly attractive $\pi^+$ and repulsive $\pi^-$ potentials in the baryon- and neutron-rich matter formed in these collisions, smaller elliptic flows are obtained for $\bar{p}$, $K^-$, and $\pi^+$ than for $p$, $K^+$, and $\pi^-$.

Also, the difference between the elliptic flows of particles and their antiparticles is found to decrease with increasing collision energy as a result of the decreasing baryon chemical potential of hadronic matter. Although our results are qualitatively consistent with the experimental observations, they somewhat underestimated the relative elliptic flow difference between $p$ and $\bar{p}$ as well as that between $\pi^-$ and $\pi^+$, and overestimated that between $K^+$ and $K^-$. In our studies, we have, however, not included other effects that may affect the $v_2$ difference between particles and their antiparticles. For example, we may have overestimated the annihilation between baryons and antibaryons, as this could be screened by other particles in hadronic matter [34]. Including the screening effect would increase the duration of the attractive potential acting on antibaryons and thus reduce their elliptic flow, leading therefore to an increase in the difference between the elliptic flows of baryons and antibaryons. Also, the different elliptic flows between particles and their antiparticles are assumed in the present study to come entirely from the hadronic mean-field potentials. As shown in Ref. [35], the collective flow of partons can also be affected by their mean-field potentials in partonic matter. If quarks and antiquarks have different mean-field potentials in partonic matter, this would then lead to different elliptic flows for particles and their antiparticles in the initial stage of the hadronic phase after hadronization. It will be of great interest to include in future studies these effects, as well as the effect due to different elliptic flows between produced and transported partons [6] and the chiral magnetic effect [7], in order to understand more quantitatively the different elliptic flows between particles and their antiparticles observed in relativistic heavy ion collisions.

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