Wind speed-up process on the windward slope of dunes in dune fields

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A R T I C L E   I N F O

Article history:
Received 1 April 2011
Received in revised form 22 October 2012
Accepted 6 November 2012
Available online 23 November 2012

Keywords:
Wind speed-up factor
Windward slope of sand dunes
Sand supply
Dune field
Sand diameter

A B S T R A C T

Wind speed-up is a key physical factor affecting the transportation of windblown sand on the windward slope of a sand dune. It contributes to the understanding of dune morphology, evolution of dune field and dunes' migration. In this study, a computational model is proposed to calculate the wind speed-up factor on the windward slope of sand dunes in a dune field. This model not only reflects the influence of neighboring dunes, but also incorporates the effect of incoming wind speed, sand diameter and thickness of sand supply which have been little considered by previous studies. The computational complexity of our model is much lower than that of the Jackson–Hunt theory and the Computational Fluid Dynamics. Based on the wind speed-up model, we investigate the probability density function of wind speed-up factor in a dune field, and give an empirical expression of wind speed-up factor, which provides a basis for accurate prediction of wind speed on the windward slope of a sand dune in a real dune field.

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1. Introduction

Wind speed-up factor has been widely used to describe the variation of wind speed along the windward slope of a sand dune [4]. It is defined as the ratio between the wind speed at the dune crest and the incoming wind speed. The recognition of wind speed-up process plays a significant role to understand the variation of windblown sand flux on the windward slope. Furthermore, it is also very important to the quantitative study of the evolution of dunes' morphology in a dune field and their migration [15,12,5]. Many dune and dune field models have considered the wind speed-up process on a windward slope, for example, Momiji et al. [7] took account of the influence of wind speed-up process on the transportation length in a cellular automaton model of a dune field. In the continuum saltation model of dune given by Sauermann [12], the wind speed-up process was treated by the Jackson–Hunt theory [2].

Field observation is a main method to verify the models of wind speed-up process [4,6,8]. Lancaster [4] carried out field observations concerning the relation between the wind speed-up factor and windward slope angle in a dune field, the results show that the wind speed-up factor increases linearly with the windward slope angle. Neumann et al. [8] carried out field observations to give a relation between the wind speed and dune morphology in the Silver Peak dune field in Clayton Valley, their results show that the wind speed-up factors range from 1.5 to 3.19. However, since wind speed is uncontrollable in field, the influence of incoming wind speed cannot be studied.

Qian et al. [10] studied the influence of incoming wind speed and windward slope angle to wind speed-up factor through wind tunnel experiments, their results show that the windward slope angle has a significant influence on the wind speed-up factor comparing with the incoming wind speed. Though many wind tunnel experiments have been conducted to reveal the influence of incoming wind speed, the wind speed-up factor obtained by wind tunnel experiments and field observations differs a lot in quantity. For example, wind tunnel experimental result is 1.9 but field observational result is 1.45 when the windward slope angle is 8°, the reason for this difference might be that the wind tunnel experiments neglected the influence of neighboring dunes in a dune field to the wind flow.

In addition, theoretical model of Momiji et al. [7] and Computational Fluid Dynamics simulations [9,11,14,3] were also applied to the study of wind speed-up process. For example, Momiji et al. [7] provided a computational formula of wind speed-up factor with consideration of dunes' migration speed and dune height. Jiang and MA [3] studied the wind field around dunes, and their results indicate that the wind speed-up factor increases linearly with windward slope angle. However, these methods were mainly carried out for single transverse dune or barchan. Therefore, though their results are consistent with field observations in quality, but differences still exist in quantity. For example, when the windward slope angle is 8°, field observation result is 1.45 [4], but the result of Computational Fluid Dynamics simulation is 2.0 [3], and the result of Momiji's theory model is about 1.1 [7]. It
suggests that due to the influence of the surrounding dunes in a dune field on the wind flow, it is unsuitable to apply the result of a single dune directly to the study of a whole dune field, and thus it is still necessary to discuss the wind speed-up process on the windward slope of dunes in a dune field in a more comprehensive way.

Above introductions demonstrate that existing studies cannot reflect the variation law of the wind speed-up factor on the windward slope of a sand dune in a dune field with the incoming wind speed, windward slope angle and dune height. It limits our understanding on the evolution of a dune field. Therefore, this study attempts to establish a computational model to model the wind speed-up process through the least square method with particular emphasis on the relation between wind speed-up factor and windward slope angle, and that between windward slope angle and dune height. Our computational results agree well with observation results, which indicate that our model can reflect the actual wind speed-up process on dune windward slope in a dune field. Then, the distribution of the wind speed-up factor in a dune field is investigated. Results show that the probability density function of wind speed-up factor satisfies a normal distribution, which changes with the evolution time, wind speed, sand diameter and sand supply. Finally, we give an empirical expression of wind speed-up factor. The model and empirical expression put forward in this study allows for an in-depth understanding of the wind speed-up process on the windward slope of a sand dune in a dune field, and thus promote the studies of windblown sand movements and the evolution of a dune field.

2. Numerical model

The computational model of wind speed-up factor is based on the variation laws of wind speed-up factor with windward slope angle and that of windward slope angle with dune height. That is, for a developing dune of $H_{x,y}$, wind speed-up factor $z_{x,y}$ is related to windward slope angle $\theta_{x,y}$, i.e., $z_{x,y} = f(\theta_{x,y})$, and $\theta_{x,y}$ is related to $H_{x,y}$, i.e., $\theta_{x,y} = g(H_{x,y})$. Here, $f(\theta_{x,y})$ expresses the relation between $z_{x,y}$ and $\theta_{x,y}$, $g(H_{x,y})$ expresses the relation between $\theta_{x,y}$ and $H_{x,y}$, in which $x, y$ represent different developing dunes. Thus, the real frictional velocity of the windward slope could be expressed as:

$$U'(x,y) = U' + \frac{U'z(x,y)}{H_{x,y}} \left\{ f[g(H_{x,y})] - 1 \right\}$$  \hspace{1cm} (1)

Here, $U'$ is the incoming wind speed, $z(x,y)$ is an arbitrary location on the windward slope.

In order to obtain $U'(x,y)$, we firstly need to determine $g(H_{x,y})$ and $f(\theta_{x,y})$. The variation of wind speed-up factor with dune height, windward slope angle and incoming wind speed are determined through wind tunnel experiments. The experiments were performed in the multi-function environmental wind tunnel of the Key Laboratory of Mechanics on Disaster and Environmental in western China, the Ministry of Education of China. Fig. 1 shows the variation of wind speed-up factor with incoming wind speed, dune height and windward slope angle. It can be found that the incoming wind speed and dune height have no significant influence on the wind speed-up factor, but the wind speed-up factor increases linearly with the windward slope angle, exactly, wind speed-up factor increases linearly with the tangent value of windward slope angle, which is consistent with existing results [4], therefore $z_{x,y}$ can be expressed by

$$z_{x,y} = A + k \tan \theta_{x,y}$$  \hspace{1cm} (2)

Here, $A$ and $k$ are fitting coefficients.

To study the variation of windward slope angle with dune height, some field observations were conducted. The windward slope angle and dune height were measured by Z-max Global positioning system, as shown in Fig. 2. The results of Morocco and Peru [1,13] were also included in Fig. 2. From Fig. 2 we can found that windward slope angle increases exponentially with dune height,

$$\theta_{x,y} = B - \ln[H_{x,y} + C]$$  \hspace{1cm} (3)

where $B$ and $C$ are fitting coefficients.

From above we can see that $A$, $B$, $C$ and $k$ are key parameters to calculate $U'(x,y)$. In this paper, we adopt the least square method to determine the $A$, $B$, $C$, and $k$, details are described as follow:

1. Determining the number of dunes $n_{x,y}$, windward slope angle $\theta_{x,y}$ and dune height $H_{x,y}$ in a dune field;
2. Using the known conditions of the height $H_{x,y}$ and the windward angle $\theta_{x,y}$ of the dune at the location $(x, y)$ as well as the height $H^\text{max}$ and the windward angle $\theta^\text{max}$ of the highest dune in the dune field, the following equations can be resolved and the parameters $(B^{(x,y)}, C^{(x,y)})$ can be written as

$$\begin{cases} C^{(x,y)} = (H^\text{max} - H_{x,y}e^{\text{max} - \theta_{x,y}})/(1 - e^{\text{max} - \theta_{x,y}}) \\ B^{(x,y)} = \text{ln}(H^\text{max} - C^{(x,y)}) \end{cases}$$  \hspace{1cm} (4)

Then, the best reasonable $(B, C)$ could be evaluated by the following equation at the smallest value,

$$r_1(x,y) = \sum_{i=1}^{n_{x,y}} [B^{(x,y)} + \ln(H_{x,y} - C^{(x,y)}) - \theta_{x,y}]^2$$  \hspace{1cm} (5)

Here, $H_{x,y}$, is the dune height corresponding to location $(x, y)$.

![Fig. 1. The variation of wind speed-up factor with incoming wind speed, dune height and windward slope angle.](image-url)
Analogously, \((A(x, y), k(x, y))\) could be got by resolving the following equations,

\[
\begin{align*}
A(x, y) &= \left(\frac{U_{\text{top}}^{\text{max}}}{U} \tan \theta_{x, y}^\text{max} - \tan \theta_{x, y}\right) \\
k(x, y) &= \left(\frac{U_{\text{top}}^{\text{max}}}{U} - k(x, y) \tan \theta_{x, y}^\text{max}\right)
\end{align*}
\]

(6)

after calculating \(\theta_{x, y}\), the velocity \((U_{\text{top}}^{\text{max}})^x\) at the corresponding dune's top and the velocity \((k(x, y))^x\) at the highest dune's top in the dune field. One of the \((A(x, y), k(x, y))\) corresponding to the smallest value of the equation,

\[
r_2(x, y) = \sum_{i=1}^{N} \left[ A(x, y) + k(x, y) \tan \theta_{x, y}^i - \frac{U_{\text{top}}^{\text{max}}}{U} \right]^2
\]

(7)

can be chosen as \((A, k)\).

For the initial time, \((U_{\text{top}}^{\text{max}})^x = U + 0.01H_{x, y}\) is obtained by field observations.

3. Results and analysis

In this study, the results obtained by our model are compared with the Momiji's theoretical method [7], together with the field observation results of Lancaster [4], as shown in Fig. 3. It shows that the result obtained by our model is consistent with the field observation qualitatively, that is, with increasing windward slope angle wind speedup factor increases, it is closer to the field observation than that obtained by Momiji's theoretical method, i.e., the relative error between our result and field observation is smaller than 10%, while the relative error between the results obtained by Momiji and the field observation is larger than 10%, whose maximum error reaches up to 28%, as shown in Table 1. Therefore, the result obtained by our model can give a better description to the actual wind speedup process in a dune field. Though our model does not consider the interaction of wind field during dunes, it reflects the influence among dunes.

In addition, based on the Comprehensive Scale-Coupled Model of Dune Field (CSCDUNE) proposed by Zheng et al. [15,16], we can obtain the probability density function of dunes' height in a dune field, which can be expressed by a normal distribution, as shown in Fig. 4, and varies with time, wind speed, sand diameter and sand supply. Thus we can get the probability density function

![Fig. 2. The variation of windward slope angle with dune height.](image)

![Fig. 3. The variation of wind speedup factor with windward slope angle.](image)

![Fig. 4. The variation of probability density function of dune size with time (\(u' = 0.5\) m/s, \(D_s = 0.3\) mm, \(H = 0.6\) m, \(S = 6000\) m x 6000 m).](image)

![Fig. 5. The variation of probability density function of wind speedup factor with time (\(u' = 0.5\) m/s).](image)

<table>
<thead>
<tr>
<th>(\delta_{r})</th>
<th>This paper</th>
<th>Momiji</th>
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<tr>
<td>8.34</td>
<td>0.0092</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 1: The relative error between Momiji's results and field observations.
of the wind speed-up factor and its change rules with time, wind speed, sand diameter and sand supply in a dune field, as shown in Fig. 5.

Fig. 5 shows that the probability density function of the wind speed-up factor in a dune field which satisfies a normal distribution, i.e.,

\[ A' \exp \left( -\frac{(h - x_c^s)^2}{2w_s^2} \right) \]  

where \( A', x_c^s \) and \( w_s \) are fitting coefficients, which respectively change with time, wind speed, sand diameter and sand supply, as shown in Figs. 6 and 7. Here, the mean value \( x_c^s \) represents the peak of the normal distribution and the variance \( w_s \) represents the distribution scope. Fig. 6 shows that the parameter \( x_c^s \) increases exponentially with time, the parameter \( A' \) decreases in a negative exponential way and \( w_s \) increases linearly with time.

At the meanwhile, the variations of \( A', x_c^s \) and \( w_s \) with sand diameter, wind speed and sand supply are shown in Figs. 7–9. It shows that with increasing wind speed, \( x_c^s \) increases linearly, \( w_s \) increases exponentially, while \( A' \) decreases linearly. It means that the larger is the wind speed, the peak location of the normal distribution is larger and the peak value is smaller. With increasing sand diameter, \( x_c^s \) decreases in a negative exponential way, \( w_s \) decreases

Fig. 6. The variation of parameters \( A', x_c^s \) and \( w_s \) with time (\( u^* = 0.5 \text{ m/s} \)).

Fig. 7. The variation of parameter \( x_c^s \) with time and sand diameter respectively (6000 m \( \times \) 6000 m).
the evolution time and of neighboring dunes in a dune field, and cannot reflect the single transverse or barchans dune which neglected the influence existing results were obtained by analyzing wind speed around a duneology, evolution of a dune field and dunes’ migration. However, the windblown sand flux on the windward slope of a sand dune, $w_s$, increases linearly. It means that the larger is the sand diameter, the peak location of normal distribution is smaller and the peak value is larger. With increasing thickness of sand supply, $\lambda_s$ and $w_s$ increase exponentially and linearly respectively, but parameter $A^*$ decreases linearly, that means the larger is the thickness of sand supply, the peak location of normal distribution is larger. Consequently, the fitting formula of parameters $A^*$, $\lambda_s$ and $w_s$ of the probability density function can be written as,

$$\begin{align*}
A^* &= 0.1 e^{-t/21} + 155 D_s - 0.02 h - 0.3 u^* + 0.2 \\
\lambda_s &= -0.1 e^{-t/14} - 343(e^{h} + 1) - 0.05 e^h + u^* \\
w_s &= 0.002 t - 127 D_s + 0.03 h + 0.5 u^* - 0.7
\end{align*}$$

Here, $D_s$ is the sand diameter, $h$ is the thickness of sand supply, $t$ is the evolution time and $u^*$ is the wind speed.

4. Conclusion

Wind speed-up factor is a key physical quantity to determine the windblown sand flux on the windward slope of a sand dune, which contributes significantly to the understanding of dune morphology, evolution of a dune field and dunes’ migration. However, existing results were obtained by analyzing wind speed around a single transverse or barchans dune which neglected the influence of neighboring dunes in a dune field, and cannot reflect the influence of the incoming wind speed, dune height and sand supply. In this paper, we proposed a computational model of wind speed-up factor, which not only can reflect the influence of neighboring dunes, but also can consider the impact of incoming wind speed, sand diameter and sand supply in the calculation of wind speed-up factor. The variation of wind speed-up factor with windward slope angle calculated by our model shows that with increasing windward slope angle the wind speedup factor increases linearly. It agrees well with the observation result both in quality and quantity which indicates that our model is better to describe the real wind field on the windward slope of a sand dune in a dune field. The computational complexity of our model is much lower than that of Jackson–Hunt theory [2] and the Computational Fluid Dynamics, which makes the simulation of the evolution of a large area of dune field more easily to be achieved.

Finally, we investigated the probability density function of the wind speed-up factor in a dune field incorporating the Comprehensive Scale-Coupled Model of Dune Field. Our results show that the probability density function of the wind speed-up factor in a dune field satisfies a normal distribution, and the variation law of the probability density function with time, wind speed, sand diameter and sand supply are given. It means that in the determination of wind speed-up factor, the influence of windward slope angle, incoming wind speed, sand diameter, the thickness of sand supply and the evolution history of a dune field should be considered.
Furthermore, we give an empirical expression of wind speed-up factor, which provides a basis for accurate prediction of wind speed on the windward slope of a sand dune in a real dune field.

**Acknowledgements**

This research was supported by a grant from the National Natural Science Foundation of China (Nos. 10972164, 11072097, 11232006, and 11202088), the Science Foundation of Ministry of Education of China (No. 308022), Fundamental Research Funds for the Central Universities (lzujbky-2009-k01) and the Project of the Ministry of Science and Technology of China (No. 2009CB421304). The authors express their sincere appreciation to the supports.

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