New delay-dependent stabilization conditions of T–S fuzzy systems with constant delay

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Abstract

This paper focuses on the problem of robust control for Takagi–Sugeno (T–S) fuzzy systems with time-delay. The delay-dependent stability analysis and controller synthesis have been addressed. The free weighting matrix method has been used for stability analysis and controller synthesis. New and less conservative delay-dependent stability conditions are proposed in terms of linear matrix inequalities (LMI). Finally, some examples are given to illustrate the effectiveness of the proposed approaches.

Keywords: T–S fuzzy systems; Time-delay; Linear matrix inequality; Robust control; Delay-dependent stability

1. Introduction

During the past two decades, the stability, performance analysis, and stabilization for Takagi–Sugeno (T–S) fuzzy systems have been studied extensively, and a lot of stability and stabilization conditions have been expressed in linear matrix inequalities (LMI) (see [17–19,21,4], and the reference therein), which can be solved numerically and effectively using convex programming techniques. However, all the aforementioned results are proposed for time-delay free T–S fuzzy systems. In practice, time-delays often occur in many dynamic systems, such as rolling systems, biological systems, metallurgical processes, network systems, and so on [15,5]. It has been shown that the existence of time-delays usually becomes the source of instability and deteriorated performance of systems. Therefore, developing some stability and stabilization conditions for the time-delay T–S fuzzy systems is important. The first work on control of T–S fuzzy systems with time-delay is proposed in [1,2] in which some LMI conditions for stability and stabilization have been proposed based on Krasovskii Lyapunov function method. In [23], stability analysis is studied for T–S fuzzy control systems with bounded uncertain delays, and an LMI-based design approach was developed. In [11], the problem of output feedback robust $H_\infty$ control problem has been discussed for T–S fuzzy systems with time-delay. Sufficient conditions for the existence of an $H_\infty$ controller are given by means of matrix inequalities. Recently, the stability conditions for interconnected T–S fuzzy systems with time-delay have been proposed in [9]. All of these results mentioned above are obtained by using delay-independent method.

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In general, there are two ways for the stability analysis and control synthesis of time-delay T–S fuzzy systems. They are delay-independent and delay-dependent approaches. For both approaches, they have their own advantages on dealing with time-delay T–S fuzzy models. The delay-independent approach provides the stability conditions irrespective of the size of the delay such as those proposed in references cited above. As a result, it may lead to comparatively conservative stability analysis results. However, the analysis procedure is usually simpler compared with the delay-dependent approach. On the other hand, the delay-dependent approach is concerned with the size of the time-delay and usually provides an upper bound of the time-delay such that the closed-loop system is stable for any time-delay less than the upper bound, see Refs. [12,16,7]. Since more information on delay is considered, it usually proposes less conservative stability analysis results compared with the delay-independent approach. However, unlike delay-independent approaches, a few results have been reported based on delay-dependent approaches. Recently, delay-dependent stabilization is first discussed in [6] for T–S fuzzy time-delay systems without uncertainties based on Lyapunov–Krasovskii function approach. The delay is assumed to be constant and unknown. A state feedback control scheme has been proposed in terms of the feasible solutions to LMIs. In [13], based on Razumikhin function approach, the delay-dependent LMI conditions for stability and stabilization have been developed for T–S fuzzy systems with state-delay and input-delay. An advantage of the approaches suggested in [13] is that the time-delay can be time-varying and non-smooth. However, since parameter uncertainty has not been considered for the stability analysis and control synthesis, the methods proposed in [6,13] will fail to work for uncertain T–S fuzzy systems with time-delay. In [20], the delay-dependent stability analysis and control synthesis have been carried out by using Lyapunov–Krasovskii function approach for uncertain T–S fuzzy systems with unknown time-varying delay. LMI-based delay-dependent conditions for robust stability and stabilization have been proposed. The results proposed in [13,20] are suitable for T–S fuzzy systems with time-varying delay. As the property of constant time-delay has not been used for the stability analysis, these approaches are more conservative when they are used to deal with the T–S fuzzy systems with constant time-delay.

In this paper, we still consider the problem of delay-dependent stability analysis and controller synthesis of T–S fuzzy systems with unknown constant delay. By using Lyapunov–Krasovskii function approach and free weighting matrix approach, new delay-dependent stability criterion and stabilization conditions will be presented in terms of LMIs. Some examples are used to illustrated the effectiveness and the feasibility of the methods proposed in this paper. The simulation results show that for dealing with T–S fuzzy systems with unknown and constant time-delay, our methods are less conservative than the existing delay-dependent approaches.

Throughout this paper, identity matrices, of appropriate dimensions, will be denoted by $I$. The notation $X > 0$ (respectively, $X \geq 0$ ), for $X \in \mathbb{R}^{n \times n}$, means that the matrix $X$ is real symmetric positive definite (respectively, positive semi-definite). If not explicitly stated, all matrices are assumed to have compatible dimensions for algebraic operations. The symbol “*” in a matrix $A \in \mathbb{R}^{n \times n}$ stands for the transposed elements in the symmetric positions.

2. Problem statement

Consider a nonlinear time-delay system represented by T–S fuzzy model as follows:

**Plant Rule $i$:**

IF $\theta_1(t)$ is $N_{i1} \cdots \theta_p(t)$ is $N_{ip}$ THEN

\[
\dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{1i} + \Delta A_{1i})x(t-\sigma) + (B_i + \Delta B_i)u(t),
\]

\[
x(t) = \varphi(t), \quad t \in [−\sigma, 0], \quad i = 1, 2, \ldots, k,
\]

where $N_{ij}$ is the fuzzy set, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input. $A_i, A_{1i} \in \mathbb{R}^{n \times n}, B_i \in \mathbb{R}^{n \times m},$ and $B_{1i} \in \mathbb{R}^{n \times r}$ are known matrices with appropriate dimensions. Scalar $k$ is the number of IF-Then rules. $\theta_1(t), \theta_2(t), \ldots, \theta_p(t)$ are the premise variables. It is assumed that the premise variables do not depend on the input $u(t)$. $\sigma > 0$ is a real positive constant representing the time-delay. The matrices $\Delta A_i, \Delta A_{1i}$ and $\Delta B_i$ denote the uncertainties in the system and take the form of

\[\Delta A_i, \Delta A_{1i}, \Delta B_i = DF(t)[E_i, E_{1i}, E_{bi}],\]

where $D, E_i, E_{1i}$, and $E_{bi}$ are known constant matrices and $F(t)$ is an unknown matrix function with the property $F^T(t)F(t) \leq I$. 
Given a pair of \((x(t)u(t))\), the final output of the fuzzy system is inferred as follows:

\[
\dot{x}(t) = \sum_{i=1}^{k} h_i(\theta(t))[(A_i + \Delta A_i)x(t) + (A_{1i} + \Delta A_{1i})x(t - \sigma) + (B_i + \Delta B_i)u(t)],
\]

\[
x(t) = \varphi(t), \quad t \in [\sigma, 0],
\]

where \(h_i(\theta(t)) = \mu_i(\theta(t))/\sum_{i=1}^{k} \mu_i(\theta(t)), \mu_i(\theta(t)) = \prod_{j=1}^{p} N_{ij}(\theta_j(t))\) and \(N_{ij}(\theta_j(t))\) is the degree of the membership of \(\theta_j(t)\) in \(N_{ij}\). In this paper, we assume that: \(\mu_i(\theta(t)) \geq 0\) for \(i = 1, 2, \ldots, k\) and \(\sum_{i=1}^{k} \mu_i(\theta(t)) > 0\) for all \(t\). Therefore, \(h_i(\theta(t)) \geq 0\) (for \(i = 1, 2, \ldots, k\)) and \(\sum_{i=1}^{k} h_i(\theta(t)) = 1\).

Based on the parallel distributed compensation, the following fuzzy control law is employed to deal with the problem of robust control for system (1).

**Control Rule i:**

IF \(\theta_1(t)\) is \(N_{11}\) \(\cdots\) \(\theta_p(t)\) is \(N_{1p}\) THEN

\[u(t) = K_i x(t), \quad i = 1, 2, \ldots, k.\]  \hspace{1cm} (2)

Hence, the overall fuzzy control law is represented by

\[u(t) = \sum_{i=1}^{k} h_i(\theta(t))K_i x(t),\]  \hspace{1cm} (3)

where \(K_i\) (\(i = 1, 2, \ldots, k\)) are the local control gains. Associated with the control law (3), the resulting closed-loop system can be expressed as follows:

\[
\dot{x}(t) = \sum_{i=1, j=1}^{k} h_i(\theta(t))h_j(\theta(t))[(A_i + \Delta A_i + (B_i + \Delta B_i)K_j)x(t) + (A_{1i} + \Delta A_{1i})x(t - \sigma)].
\]  \hspace{1cm} (4)

Before stating our main results, the following lemmas are first presented, which will be used in the proofs of our results.

**Lemma 1** (Wang et al. [22]). Let \(A, D, E, F(t)\) and \(P\) be real matrices of appropriate dimensions with \(P > 0\) and \(F(t)\) satisfying

\[F^T(t)F(t) \leq I.\]

Then, the following inequality holds for any \(\varepsilon > 0\),

\[DF(t)E + E^T F^T(t)D^T \leq \varepsilon DD^T + \varepsilon^{-1}E^TE.\]

**Lemma 2** (Kwon and Park [10]). For any constant matrix \(M > 0\), any scalar \(\sigma > 0\), and any vector function \(x(t): [0, \sigma] \rightarrow \mathbb{R}^n\) such that the integrals concerned are well defined, the following inequality holds:

\[
\left[ \int_{t-\sigma}^{t} x(s) \, ds \right]^T M \left[ \int_{t-\sigma}^{t} x(s) \, ds \right] \leq \sigma \int_{t-\sigma}^{t} x^T(s) M x(s) \, ds.
\]

3. Delay-dependent robust stability criteria

In this section, the problem of robust stability performance analysis for system (4) is addressed. For the stability analysis of system (4), it is assumed that the feedback gain matrices \(K_i\) have been well designed.

**Theorem 1.** Given a scalar \(\sigma_0 > 0\). System (4) is robustly asymptotically stable for any constant time-delay \(\sigma\) satisfying \(0 \leq \sigma \leq \sigma_0\) if there exist matrices \(P > 0, P_1 > 0, T_1 > 0\) and \(T_2 > 0\), as well as some matrices \(M_1, M_2, M_3\) and \(M_4\),
and positive scalars $z_{ii} > 0$ and $z_{ij} > 0$ such that the following LMIs hold for $i, j = 1, 2, \ldots, k (i \leq j)$.

\[
\begin{bmatrix}
\Phi_{11}(i, i) & \Phi_{12}(i, i) & \Phi_{13}(i, i) & -M_1 + M_4^T & PD \\
* & \Phi_{22}(i, i) & A_i^T P_1 - M_3^T & -M_2 - M_4^T & 0 \\
* & * & \sigma_0 T_2 - 2 P_1 & -M_3 & P_1 D \\
* & * & * & -\frac{1}{\sigma_0} T_2 - M_4 - M_4^T & 0 \\
* & * & * & * & -z_{ij} I
\end{bmatrix} < 0,
\]

(5)

Where

\[
\Phi_{11}(i, j) = P \frac{(A_i + A_j + B_i K_j + B_j K_i)^T}{2} + \frac{(A_i + A_j + B_i K_j + B_j K_i)^T}{2} P \\
+ T_1 + M_1 + M_4^T + \frac{\tilde{E}_{ij} + \tilde{E}_{ij}^T}{2},
\]

\[
\Phi_{12}(i, j) = P \frac{(A_i + A_j)^T}{2} - M_1 + M_4^T + \frac{\tilde{E}_{ij} + \tilde{E}_{ij}^T}{2} \left( \frac{E_{ii} + E_{ij}}{2} \right),
\]

\[
\Phi_{13}(i, j) = \frac{(A_i + A_j + B_i K_j + B_j K_i)^T}{2} P_1 + M_3^T,
\]

\[
\Phi_{22}(i, j) = -T_1 - M_2 - M_4^T + z_{ij} \left( \frac{E_{ii} + E_{ij}}{2} \right)^T \left( \frac{E_{ii} + E_{ij}}{2} \right),
\]

\[
\tilde{E}_{ij} = E_i + E_{bi} K_j + E_j + E_{bj} K_i.
\]

**Proof.** In the context, the following notations will be used for simplicity:

\[
\tilde{A}_{ij} = A_{ij} + \Delta A_{ij}, \quad A_{ij} = A_i + B_i K_j, \quad \Delta A_{ij} = \Delta A_i + \Delta B_i K_j,
\]

\[
\tilde{A}_{li} = A_{li} + \Delta A_{li}, \quad h_i = h_i(\theta(t)), \quad x_{\sigma} = x(t - \sigma), \quad w = w(t).
\]

By using these notations, system (4) can be rewritten in the following form:

\[
\dot{x} = \sum_{i,j=1}^{k} h_i h_j \left( \frac{\tilde{A}_{ij} + \tilde{A}_{ij}}{2} x + \frac{\tilde{A}_{li} + \tilde{A}_{lj}}{2} x_{\sigma} \right).
\]

(7)

Consider the following Lyapunov function candidate:

\[
V = x^T P x + \int_{t-\sigma}^{t} x^T(s) T_1 x(s) \, ds + \int_{t-\sigma_0}^{t} (s - t + \sigma_0) \dot{x}^T(s) T_2 \dot{x}(s) \, ds.
\]

(8)
Differentiating $V$ along the trajectory of (4) yields

$$
\dot{V} = 2x^T P \left[ \sum_{i,j=1}^{k} h_i h_j (\ddot{A}_{ij}x + \ddot{A}_{il} x_{l0}) \right] + x^T T_1 x - x_{0}^T T_1 x_{0} + \sigma_0 \dot{x}^T T_2 \dot{x} - \int_{t-\sigma_0}^{t} \dot{x}^T (s) T_2 \dot{x}(s) \, ds
$$

$$
= 2x^T P \left[ \sum_{i,j=1}^{k} h_i h_j \left( \frac{\ddot{A}_{ij} + \ddot{A}_{jl}}{2} x + \frac{\ddot{A}_{il} + \ddot{A}_{lj}}{2} x_{l0} \right) \right] + x^T T_1 x
$$

$$
- x_{0}^T T_1 x_{0} + \sigma_0 \dot{x}^T T_2 \dot{x} - \int_{t-\sigma_0}^{t} \dot{x}^T (s) T_2 \dot{x}(s) \, ds
$$

$$
= \sum_{i,j=1}^{k} h_i h_j \left[ x^T \left( \frac{P \ddot{A}_{ij} + \ddot{A}_{jl}}{2} x + \frac{P \ddot{A}_{il} + \ddot{A}_{lj}}{2} x_{l0} \right) \right] + 2x^T P \frac{\ddot{A}_{il} + \ddot{A}_{lj}}{2} x_{l0}
$$

$$
- x_{0}^T T_1 x_{0} + \sigma_0 \dot{x}^T T_2 \dot{x} - \int_{t-\sigma_0}^{t} \dot{x}^T (s) T_2 \dot{x}(s) \, ds.
$$

(9)

Note that $0 \leq \sigma \leq \sigma_0$ and $T_2 > 0$. Applying Lemma 2 to the term $- \int_{t-\sigma_0}^{t} \dot{x}^T (s) T_2 \dot{x}(s) \, ds$ gives the following inequality:

$$
- \int_{t-\sigma_0}^{t} \dot{x}^T (s) T_2 \dot{x}(s) \, ds \leq - \int_{t-\sigma}^{t} \dot{x}^T (s) T_2 \dot{x}(s) \, ds
$$

$$
\leq - \frac{1}{\sigma} \left[ \int_{t-\sigma_0}^{t} \dot{x}(s) \, ds \right]^T T_2 \left[ \int_{t-\sigma}^{t} \dot{x}(s) \, ds \right]
$$

$$
\leq - \frac{1}{\sigma_0} \left[ \int_{t-\sigma_0}^{t} \dot{x}(s) \, ds \right]^T T_2 \left[ \int_{t-\sigma}^{t} \dot{x}(s) \, ds \right].
$$

(10)

Now, define $e^T = [x^T x_{0}^T \dot{x}^T \int_{t-\sigma}^{t} \dot{x}^T]$. Taking (9) and (10) into account, the following inequality can be obtained:

$$
\dot{V} \leq \sum_{i,j=1}^{k} h_i h_j e^T \begin{bmatrix}
    P \ddot{A}_{ij} + \ddot{A}_{jl} & \frac{P \ddot{A}_{il} + \ddot{A}_{lj}}{2} P + T_1 & \frac{P \ddot{A}_{il} + \ddot{A}_{lj}}{2} & 0 & 0 \\
    * & -T_1 & 0 & 0 & e.
\end{bmatrix}
$$

(11)

Define a matrix $M^T = [M_1^T M_2^T M_3^T M_4^T]$ with appropriate dimensions. Then, we have

$$
0 = 2e^T M \left[ x - x_{0} - \int_{t_0}^{t} \dot{x} \, ds \right]
$$

$$
= 2e^T M \left[ I - I \right] e
$$

$$
= e^T \begin{bmatrix}
    M_1 + M_1^T & -M_1 + M_2^T & M_3^T & -M_1 + M_4^T \\
    * & -M_2 + M_2^T & -M_3^T & -M_2 - M_4^T \\
    * & * & 0 & -M_3 \\
    * & * & * & -M_4 - M_4^T
\end{bmatrix} e.
$$

(12)
In addition, for any matrix \( P_1 > 0 \), it follows from (7) that
\[
0 = 2\dot{x}^T P_1 \sum_{i,j=1}^{k} h_i h_j \left( \frac{\tilde{A}_{ij} + \tilde{A}_{ij}}{2} x + \frac{\tilde{A}_{1i} + \tilde{A}_{1j}}{2} x_{\sigma} \right) - 2\dot{x}^T P_1 \dot{x}
\]
\[
= \sum_{i,j=1}^{k} h_i h_j e^T \begin{bmatrix} 0 & 0 & \frac{\tilde{A}_{ij} + \tilde{A}_{ji}}{2} P_1 & 0 \\ 0 & 0 & \frac{\tilde{A}_{ij} + \tilde{A}_{ji}}{2} P_1 & 0 \\ * & * & -2P_1 & 0 \\ * & * & * & 0 \end{bmatrix} e. \tag{13}
\]

Consequently, adding (12) and (13) to (11) results in the inequality below:
\[
\dot{V} \leq \sum_{i,j=1}^{k} h_i h_j e^T [\Pi(i, j) + \Delta\Pi(i, j)] e
\]
\[
= \sum_{i=1}^{k} h_i^2 e^T [\Pi(i, i) + \Delta\Pi(i, i)] e + \sum_{i,j=1, i \neq j}^{k} h_i h_j e^T [\Pi(i, j) + \Delta\Pi(i, j)] e, \tag{14}
\]

where
\[
\Pi(i, j) = \begin{bmatrix} \theta_{11ij} + \theta_{11ji} & \theta_{12ij} + \theta_{12ji} & \frac{\theta_{13ij} + \theta_{13ji}}{2} & \theta_{14ij} + \theta_{14ji} \\ \frac{2}{2} & \theta_{22ij} + \theta_{22ji} & \frac{\theta_{23ij} + \theta_{23ji}}{2} & \theta_{24ij} + \theta_{24ji} \\ * & \frac{\theta_{23ij} + \theta_{23ji}}{2} & \theta_{33ij} + \theta_{33ji} & \frac{\theta_{34ij} + \theta_{34ji}}{2} \\ * & * & * & \frac{\theta_{44ij} + \theta_{44ji}}{2} \end{bmatrix},
\]
\[
\Delta\Pi(i, j) = \begin{bmatrix} \frac{\Delta\theta_{11ij} + \Delta\theta_{11ji}}{2} & \frac{\Delta\theta_{12ij} + \Delta\theta_{12ji}}{2} & \frac{\Delta\theta_{13ij} + \Delta\theta_{13ji}}{2} & 0 \\ \frac{\Delta\theta_{22ij} + \Delta\theta_{22ji}}{2} & \frac{\Delta\theta_{23ij} + \Delta\theta_{23ji}}{2} & 0 & 0 \\ * & 0 & \frac{2}{2} & 0 \\ * & * & * & 0 \end{bmatrix},
\]

\[
\theta_{11ij} = PA_{ij} + A_{ij}^T P + T_1 + M_1 + M_1^T, \\
\Delta\theta_{11ij} = P \Delta A_{ij} + \Delta A_{ij}^T P,
\]
\[
\theta_{12ij} = PA_{1ij} - M_1 + M_2^T, \quad \Delta\theta_{12ij} = P \Delta A_{1i},
\]
\[
\theta_{13ij} = A_{ij}^T P_1 + M_3^T, \quad \Delta\theta_{13ij} = -\Delta A_{ij}^T P_1,
\]
\[
\theta_{14ij} = -M_1 + M_4^T, \quad \Delta\theta_{14ij} = 0,
\]
\[
\theta_{22ij} = -T_1 - M_2 - M_2^T, \quad \Delta\theta_{22ij} = 0,
\]
\[
\theta_{23ij} = A_{1ij}^T P_1 - M_3^T, \quad \Delta\theta_{23ij} = \Delta A_{1ij}^T P_1,
\]
\[
\theta_{24ij} = -M_2 - M_4^T, \quad \Delta\theta_{24ij} = 0.
\]
\[ \theta_{33ij} = \sigma_0 T_2 - 2P_1, \quad \theta_{34ij} = -M_3, \]
\[ \theta_{44ij} = -\frac{1}{\sigma_0} T_2 - M_4 - M_4^T. \]

Here, the matrices \( M_i \) (\( i = 1, 2, \ldots, 4 \)) and \( P_1 \) are the so-called free weighting matrices [8,14]. By using the free weighting matrix method, two zero equalities (12) and (13) are developed. Then, these two zero equalities are added to (11) to establish new delay-dependent stability criterion. Such a method is called “free weighting matrix method”. Application of the free weighting matrix method results in less conservative delay-dependent conditions.

Note that \( \tilde{E}_{ij} = E_i + E_{bi} K_j + E_j + E_{bj} K_i \) and \( [\Delta A_i, \Delta A_{ii}, \Delta B_{ij}] = DF(t)[E_i, E_{ii}, E_{bi}] \). \( \Delta II(i, j) \) can be expressed in the following form:

\[
\Delta II(i, j) = \begin{bmatrix} P & 0 & 0 & PDD^TP_1 \\ 0 & \frac{\tilde{E}_{ij}}{2} + \frac{E_{ii} + E_{jj}}{2} & 0 & 0 \\ P_1 & 0 & 0 & 0 \\ 0 & 0 & P DD^TP_1 & 0 \end{bmatrix} + \epsilon^{-1}_{ij} \begin{bmatrix} \tilde{E}_{ij} \tilde{E}_{ij} \\ 0 \\ P DD^TP_1 \\ 0 \end{bmatrix} E_{ij} + \epsilon^{-1}_{ij} \begin{bmatrix} P \phi_{11ii} P \phi_{12ii} P \phi_{13ii} P \phi_{14ii} \\ 0 \\ P DD^TP_1 \\ 0 \end{bmatrix} e, \quad (15)
\]

where \( 0_{i\times j} \) stands for the zero matrix with \( i \) rows and \( j \) columns. Applying Lemma 1 to (15) produces the following inequality:

\[
\Delta II(i, j) \leq \epsilon_{ij} \begin{bmatrix} P DD^TP & 0 & 0 & P DD^TP_1 \\ 0 & \frac{\tilde{E}_{ij}}{2} + \frac{E_{ii} + E_{jj}}{2} & 0 & 0 \\ * & 0 & 0 & 0 \\ * & P DD^TP_1 & 0 & 0 \end{bmatrix} + \epsilon^{-1}_{ij} \begin{bmatrix} \tilde{E}_{ij} \tilde{E}_{ij} \\ 0 \\ 0 \\ 0 \end{bmatrix} E_{ij} + \epsilon^{-1}_{ij} \begin{bmatrix} P \phi_{11ii} P \phi_{12ii} P \phi_{13ii} P \phi_{14ii} \\ 0 \\ 0 \\ 0 \end{bmatrix} e, \quad (16)
\]

Apparently, substituting (16) into (14) shows that

\[
\dot{V} \leq \sum_{i=1}^{k} h_i^2 e^T \begin{bmatrix} \phi_{11ii} & \phi_{12ii} & \phi_{13ii} & \phi_{14ii} \\ * & \phi_{22ii} & \phi_{23ii} & \phi_{24ii} \\ * & * & \phi_{33ii} & \phi_{34ii} \\ * & * & * & \phi_{44ii} \end{bmatrix} e + \sum_{i,j=1,i\neq j}^{k} h_i h_j e^T \begin{bmatrix} \phi_{11ij} & \phi_{12ij} & \phi_{13ij} & \phi_{14ij} \\ * & \phi_{22ij} & \phi_{23ij} & \phi_{24ij} \\ * & * & \phi_{33ij} & \phi_{34ij} \\ * & * & * & \phi_{44ij} \end{bmatrix} e, \quad (17)
\]

where \( \epsilon_{ij} > 0, \ i, j = 1, 2, \ldots, k \) \((i \leq j)\) are positive constants, and

\[
\phi_{11ij} = P \frac{(A_i + A_j + B_i K_j + B_j K_i)^T}{2} + \frac{(A_i + A_j + B_i K_j + B_j K_i)^T}{2} + T_1 + M_1 + M_1^T + \epsilon_{ij} P DD^TP_1 + \epsilon^{-1}_{ij} \tilde{E}_{ij} \tilde{E}_{ij}/2, 
\]
\[
\phi_{12ij} = P \frac{(A_{ii} + A_{jj})}{2} - M_1 + M_1^T + \epsilon_{ij} P DD^TP + \epsilon^{-1}_{ij} \tilde{E}_{ij} \tilde{E}_{ij}/2, 
\]
\[
\phi_{13ij} = P \frac{(A_i + A_j + B_i K_j + B_j K_i)^T}{2} - M_2 + M_2^T + \epsilon_{ij} P DD^TP_1, 
\]
\[
\phi_{14ij} = -M_1 + M_1^T, 
\]
\[
\phi_{22ij} = -T_1 - M_2 - M_2^T + \epsilon^{-1}_{ij} \left( \frac{E_{ii} + E_{jj}}{2} \right) \left( \frac{E_{ii} + E_{jj}}{2} \right), 
\]
\[
\begin{align*}
\phi_{23ij} = & \frac{(A_{1i} + A_{1j})^T}{2} P_1 - M_3^T, \\
\theta_{24ij} = & -M_2 - M_4^T, \\
\phi_{33ij} = & \sigma T_2 - 2P_1 + \varepsilon_{ij} P_1 D D^T P_1, \\
\phi_{34ij} = & -M_3, \\
\phi_{44ij} = & -\frac{1}{\sigma} T_2 - M_4 - M_4^T.
\end{align*}
\]

Let \( z_{ij} = \varepsilon_{ij}^{-1} \). By using Schur complement to (5) and (6), it can be easily proved that for \( i, j = 1, 2, \ldots, k \ (i \leq j) \)

\[
\begin{bmatrix}
\phi_{11ij} & \phi_{12ij} & \phi_{13ij} & \phi_{14ij} \\
* & \phi_{22ij} & \phi_{23ij} & \phi_{24ij} \\
* & * & \phi_{33ij} & \phi_{34ij} \\
* & * & * & \phi_{44ij}
\end{bmatrix} < 0. \tag{18}
\]

Furthermore, (17) and (18) imply that \( \dot{V} < 0 \), which means that system (4) is robustly stable for any \( \sigma \) satisfying \( 0 \leq \sigma \leq \sigma_0 \). The proof is thus completed. \( \square \)

Theorem 1 presents a delay-dependent robust stability criterion for system (4) in the LMI form. It can be clearly seen that by choosing \( K_i = 0 \) in Theorem 1, a robust stability criterion for the unforced system (4) can be obtained as follows.

**Corollary 1.** Given a scalar \( \sigma_0 > 0 \). System (4) with \( u(t) \equiv 0 \) is robustly asymptotically stable for any constant time-delay \( \sigma \) satisfying \( 0 \leq \sigma \leq \sigma_0 \) if there exist matrices \( P > 0, P_1 > 0, T_1 > 0 \) and \( T_2 > 0 \), as well as some matrices \( M_1, M_2, M_3 \) and \( M_4 \), and positive scalars \( \varepsilon_i > 0 \) such that the following LMIs hold for \( i = 1, 2, \ldots, k \):

\[
\begin{bmatrix}
\Phi_{11(i)} & \Phi_{12(i)} & A_{1i}^T P_1 + M_3^T & -M_1 + M_4^T & PD \\
* & \Phi_{22(i)} & A_{1i}^T P_1 - M_3^T & -M_2 - M_4^T & 0 \\
* & * & \sigma_0 T_2 - 2P_1 & -M_3 & P_1 D \\
* & * & * & -\frac{1}{\sigma_0} T_2 - M_4 - M_4^T & 0 \\
* & * & * & * & -\varepsilon_i I
\end{bmatrix} < 0, \tag{19}
\]

where

\[
\begin{align*}
\Phi_{11(i)} &= PA_i + A_{1i}^T P + T_1 + M_1 + M_1^T + \varepsilon_i E_i^T E_i, \\
\Phi_{12(i)} &= PA_{1i} - M_1 + M_2^T + \varepsilon_i E_i^T E_{1i}, \\
\Phi_{22(i)} &= -T_1 - M_2 - M_2^T + \varepsilon_i E_{1i}^T E_{1i}.
\end{align*}
\]

4. Fuzzy controller design

In this section, we will develop a fuzzy controller design scheme for system (4) based on the results of the last section. Obviously, the main objective is to determine the feedback gain matrices \( K_i \) such that inequality (18) holds. To this end, let us choose \( P_1 = dP \) with \( d \) being a design parameter in (18). Pre- and post-multiplying (18) by \( \text{diag}(X X X) \)
with \( X = P^{-1} \), it follows that for \( i, j = 1, 2, \ldots, k \) \((i \leq j)\)

\[
\begin{bmatrix}
\tilde{\phi}_{11ij} & \tilde{\phi}_{12ij} & \tilde{\phi}_{13ij} & \tilde{\phi}_{14ij} \\
* & \tilde{\phi}_{22ij} & \tilde{\phi}_{23ij} & \tilde{\phi}_{24ij} \\
* & * & \tilde{\phi}_{33ij} & \tilde{\phi}_{34ij} \\
* & * & * & \tilde{\phi}_{44ij}
\end{bmatrix} < 0,
\]

where

\[
\begin{align*}
\tilde{\phi}_{11ij} &= \frac{(A_i + A_j)X + B_iN_j + B_jN_i}{2} X + \frac{X(A_i + A_j)^T + (B_iN_j + B_jN_i)^T}{2} \\
&\quad + S_i + G_1 + G_1^T + \varepsilon_{ij}DD^T + \varepsilon_{ij}^{-1} \frac{X \tilde{E}_{ij} \tilde{E}_{ij}^T}{2}, \\
\tilde{\phi}_{12ij} &= \frac{(A_{1i} + A_{1j})X}{2} - G_1 + G_2^T + \varepsilon_{ij}^{-1} \frac{X \tilde{E}_{ij} \tilde{E}_{ij}^T}{2} \left( \frac{E_{1i} + E_{1j}}{2} \right) X, \\
\tilde{\phi}_{13ij} &= dX \frac{(A_{1i} + A_{1j})^T + (B_iN_j + B_jN_i)^T}{2} + G_3^T + \varepsilon_{ij} dDD^T, \\
\tilde{\phi}_{14ij} &= -G_1 + G_4^T, \\
\tilde{\phi}_{22ij} &= -S_1 - G_2 - G_2^T + \varepsilon_{ij}^{-1} X \left( \frac{E_{1i} + E_{1j}}{2} \right)^T \left( \frac{E_{1i} + E_{1j}}{2} \right) X, \\
\tilde{\phi}_{23ij} &= -G_3, \\
\tilde{\phi}_{24ij} &= -G_4, \\
\tilde{\phi}_{33ij} &= \sigma S_2 - 2dX + \varepsilon_{ij} d^2 DD^T, \\
\tilde{\phi}_{34ij} &= -G_3, \\
\tilde{\phi}_{44ij} &= -\frac{1}{\sigma} S_2 - G_4 - G_4^T,
\end{align*}
\]

with \( S_i = XT_i X, G_i = XM_i X, \) and \( N_i = K_i X \). By applying Schur complement to (20) for \( i, j = 1, 2, \ldots, k \) and \( i < j = 1, 2, \ldots, k \), we get the following result.

**Theorem 2.** Given scalars \( \sigma_0 > 0 \) and \( d \). If there exist matrices \( P > 0, S_1 > 0 \) and \( S_2 > 0 \), as well as some matrices \( G_i (i = 1, 2, 3, 4) \), and positive scalars \( \varepsilon_{ii} > 0 \) and \( \varepsilon_{ij} > 0 \) satisfying the following LMIs for \( i, j = 1, 2, \ldots, k \) \((i \leq j)\):

\[
\begin{bmatrix}
\Phi_{11ii} & \Phi_{12ii} & \Phi_{13ii} & -G_1 + G_4^T & XE_i^T + N_i^T E_{bi}^T \\
* & -S_1 - G_2 - G_2^T & \Phi_{23ii} & -G_2 - G_4^T & XE_i^T \\
* & * & \Phi_{33ii} & -G_3 & 0 \\
* & * & * & -\frac{1}{\sigma_0} S_2 - G_4 - G_4^T & 0 \\
* & * & * & * & -\varepsilon_{ii} I
\end{bmatrix} < 0
\]

(21)
and

\[
\begin{bmatrix}
\Phi_{11ij} & \Phi_{12ij} & \Phi_{13ij} & -G_1 + G_4^T & \frac{\dot{E}_{ij}^T}{2} \\
\ast & -S_1 - G_2 - G_2^T & \Phi_{23ij} & -G_2 - G_4^T & X \left( \frac{E_{ij} + E_{ij}}{2} \right)^T \\
\ast & \ast & \Phi_{33ij} & -G_3 & 0 \\
\ast & \ast & \ast & -\frac{1}{\sigma_0} S_2 - G_4^T & 0 \\
\ast & \ast & \ast & \ast & -\epsilon_{ij} I
\end{bmatrix} < 0.
\] (22)

where

\[
\Phi_{11ij} = \frac{(A_i + A_j)X + B_i N_j + B_j N_i}{2} + \frac{X(A_i + A_j)^T + (B_i N_j + B_j N_i)^T}{2}
\]

\[
+S_1 + G_1 + G_1^T + \epsilon_{ij} DD^T,
\]

\[
\Phi_{12ij} = \frac{(A_{1i} + A_{1j})X}{2} - G_1 + G_2^T,
\]

\[
\Phi_{13ij} = d X \left( \frac{A_i + A_j}{2} \right)^T + \frac{(B_i N_j + B_j N_i)^T}{2} + G_3^T + \epsilon_{ij} d DD^T,
\]

\[
\Phi_{23ij} = d X \left( \frac{A_{1i} + A_{1j}}{2} \right)^T - G_3^T,
\]

\[
\Phi_{33ij} = \sigma_0 S_2 - 2d X + \epsilon_{ij} d^2 DD^T
\]

then the closed-loop system composed of (4) and (3) is robustly asymptotically stable for any constant time-delay \( \sigma \) satisfying \( 0 \leq \sigma \leq \sigma_0 \). Moreover, the feedback gain matrices \( K_i \) are given by

\[ K_i = N_i X^{-1}, \quad i = 1, 2, \ldots, k. \]

**Remark 1.** In Theorem 2, \( d \) is a parameter chosen by designers. The simulations show that the smaller the \( d \), the larger the feedback gains. However, we have not found a way to choose an optimum \( d \) for the controller design. It is difficult to find such an optimum \( d \) for the controller design.

5. Computer simulation

In this section, three examples are used to illustrate the effectiveness and merits of the results proposed in this paper. The first example is taken from [13], which is delay-dependently stable. Using this example, we show the advantages of our delay-dependent stability criterion. The second and the third are, respectively, from [20] and [3]. They are used to show how to design the robust fuzzy controller by using the proposed method.

**Example 1.** Consider the following T–S fuzzy system [13]:

\[
\dot{x} = \sum_{i=1}^{2} h_i(A_i x(t) + A_{i1} x(t - \sigma))
\] (23)

with

\[
A_1 = \begin{bmatrix}
-2.1 & 0.1 \\
-0.2 & -0.9
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
-1.9 & 0 \\
-0.2 & -1.1
\end{bmatrix},
\]

\[
A_{11} = \begin{bmatrix}
-1.1 & 0.1 \\
-0.8 & -0.9
\end{bmatrix}, \quad A_{21} = \begin{bmatrix}
-0.9 & 0 \\
-1.1 & -1.2
\end{bmatrix}.
\]
Here, it is assumed that time-delay $\sigma$ is an unknown constant. The fuzzy membership functions are taken as $h_1 = \sin^2(x_1 + 0.5)$ and $h_2 = \cos^2(x_1 + 0.5)$. The simulation result shows that system (23) is asymptotically stable for the maximum delay $\sigma = 6.5$, see Fig. 1. By using Theorem 3.1 in [13], the maximum allowable value of $\sigma$ is 0.65. Applying Theorem 1 in [6] gives a maximum allowable value of $\sigma = 1.25$, and the Lemma 1 in [20] suggests the maximum allowable value of $\sigma = 1.85$. However, by using Corollary 1 in this paper with $d = 0.5$, the maximum allowable value is $\sigma = 3.15$.

**Example 2.** Consider the following T–S fuzzy system [20]:

$$
\dot{x} = \sum_{i=1}^{2} h_i ((A_i + \Delta A_1)x(t) + (A_{i1} + \Delta A_{i1})x(t - \sigma) + B_iu(t))
$$

(24)

with

$$
A_1 = \begin{bmatrix} 0 & 1 \\ 0.1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 0 & -0.5 - 1.5 \beta \end{bmatrix},
$$

$$
A_{11} = A_{21} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & -0.2 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
$$

$$
\Delta A_i = DF(t)E_i, \quad \Delta A_{i1} = DF(t)E_{i1},
$$

$$
D = \begin{bmatrix} -0.03 & 0 \\ 0 & 0.03 \end{bmatrix}, \quad E_1 = E_2 = \begin{bmatrix} -0.15 & 0.2 \\ 0 & 0.04 \end{bmatrix},
$$

$$
E_{11} = E_{12} = \begin{bmatrix} -0.05 & -0.35 \\ 0.08 & -0.45 \end{bmatrix}, \quad \beta = \frac{0.01}{\pi}.
$$

and $h_1 = (1 - \frac{1}{1+\exp(-3(x_2/0.5 - \pi/2))}) \times (1+\exp(-3(x_2/0.5 + \pi/2)))$, $h_2 = 1 - h_1$. For the case of $\sigma$ being constant and unknown and $\Delta A_i = \Delta A_{i1} = 0$, the existing delay-dependent approaches are used to design the fuzzy controllers,
and the corresponding results are shown as follows.

<table>
<thead>
<tr>
<th>Paper</th>
<th>$\sigma_{\text{Max}}$</th>
<th>$K_1$</th>
<th>$K_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[6]</td>
<td>3.4385</td>
<td>[-0.9390 0.3123]</td>
<td>[-0.3277 -0.3636]</td>
</tr>
<tr>
<td>This paper</td>
<td>25.7865</td>
<td>[-1.2141 0.8750]</td>
<td>[-1.2141 -0.6202]</td>
</tr>
</tbody>
</table>

For the case of $\Delta A_i \neq 0$ and $\Delta A_{i1} \neq 0$, the approaches proposed in [13,6] cannot be used to design feedback controllers as the system contains the uncertainties. The method in [20] and Theorem 2 with $d = 0.5$ can be used to design the fuzzy controllers. The corresponding results are listed below.

<table>
<thead>
<tr>
<th>Paper</th>
<th>$\sigma_{\text{Max}}$</th>
<th>$K_1$</th>
<th>$K_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This paper</td>
<td>20.5640</td>
<td>[-1.3987 -0.6601]</td>
<td>[-1.3991 -2.1607]</td>
</tr>
</tbody>
</table>

The simulation was run under the initial condition $x(0) = [3 - 1]^T$ and uncertainty $F(t) = \text{diag}[\sin t \ \cos t]$. Figs. 2 and 3 show the simulation results. From the simulation results, it can be clearly seen that the proposed control law $u(t) = h_1[-1.3987 - 0.6601]x + h_2[-1.3991 - 2.1607]x$ guarantees the asymptotic stability of the closed-loop system.

**Example 3.** Consider the nonlinear system with time-delay [20].

\[
\begin{align*}
\dot{x}_1(t) &= -a \frac{v \bar{\tau}}{(L + \Delta L(t))t_0} x_1(t) - (1-a) \frac{v \bar{\tau}}{(L + \Delta L(t))t_0} x_1(t - \sigma) + \frac{v \bar{\tau}}{(l + \Delta l(t))t_0} u(t), \\
\dot{x}_2(t) &= \frac{v \bar{\tau}}{(L + \Delta L(t))t_0} x_1(t) + (1-a) \frac{v \bar{\tau}}{(L + \Delta L(t))t_0} x_1(t - \sigma), \\
\dot{x}_3(t) &= \frac{v \bar{\tau}}{l_0} \sin \left( x_2(t) + \frac{v \bar{\tau}}{2(L + \Delta L(t))} x_1(t) + (1-a) \frac{v \bar{\tau}}{2(L + \Delta L(t))} x_1(t - \sigma) \right),
\end{align*}
\]

where $a = 0.7$, $v = -1.0$, $\bar{t} = 2.0$, $t_0 = 0.5$, $L = 5.5$, $l = 2.8$, $-0.2619 \leq \Delta L \leq 0.2895$ and $-0.1333 \leq \Delta l \leq 0.1474$ (these inequalities imply that $0.95 \frac{1}{L} \leq \frac{1}{L+\Delta L} \leq 1.05 \frac{1}{L}$ and $0.95 \frac{1}{l} \leq \frac{1}{l+\Delta l} \leq 1.05 \frac{1}{l}$). The following fuzzy rules
are employed:

Rule 1: IF \( \theta(t) = x_2(t) + a \frac{v^2}{2L} x_1(t) + (1 - a) \frac{v^2}{2L} x_1(t - \sigma) \) is about 0, THEN

\[
\dot{x}(t) = (A_1 + \Delta A_1)x(t) + (A_{11} + \Delta A_{11})x(t - \sigma) + (B_1 + \Delta B_1)u(t).
\]

Rule 2: IF \( \theta(t) = x_2(t) + a \frac{v^2}{2L} x_1(t) + (1 - a) \frac{v^2}{2L} x_1(t - \sigma) \) is about \( \pi \) or \(-\pi\), THEN

\[
\dot{x}(t) = (A_2 + \Delta A_2)x(t) + (A_{21} + \Delta A_{21})x(t - \sigma) + (B_2 + \Delta B_2)u(t),
\]

where

\[
A_1 = \begin{bmatrix}
-a \frac{v\bar{t}}{L_{l0}} & 0 & 0 \\
-a \frac{v\bar{t}}{L_{l0}} & 0 & 0 \\
-v^2 \frac{t^2}{2L_{l0}} & v\bar{t} & 0
\end{bmatrix}, \quad A_{11} = \begin{bmatrix}
-(1 - a) \frac{v\bar{t}}{L_{l0}} & 0 & 0 \\
(1 - a) \frac{v\bar{t}}{L_{l0}} & 0 & 0 \\
(1 - a) \frac{v^2 t^2}{2L_{l0}} & 0 & 0
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
\frac{v\bar{t}}{L_{l0}} \\
0 \\
0
\end{bmatrix},
\]

\[
A_2 = \begin{bmatrix}
-a \frac{v\bar{t}}{L_{l0}} & 0 & 0 \\
-a \frac{v\bar{t}}{L_{l0}} & 0 & 0 \\
-v^2 \frac{t^2}{2L_{l0}} & \frac{d v\bar{t}}{L_{l0}} & 0
\end{bmatrix}, \quad A_{21} = \begin{bmatrix}
-(1 - a) \frac{v\bar{t}}{L_{l0}} & 0 & 0 \\
(1 - a) \frac{v\bar{t}}{L_{l0}} & 0 & 0 \\
(1 - a) \frac{v^2 t^2}{2L_{l0}} & 0 & 0
\end{bmatrix}, \quad B_2 = \begin{bmatrix}
\frac{v\bar{t}}{L_{l0}} \\
0 \\
0
\end{bmatrix},
\]

\[
\Delta A_1 = 0.05 \delta(t) \begin{bmatrix}
0.5091 & 0 & 0 \\
-0.5091 & 0 & 0 \\
0.5091 & 0 & 0
\end{bmatrix}, \quad \Delta A_{11} = 0.05 \delta(t) \begin{bmatrix}
0.2182 & 0 & 0 \\
-0.2182 & 0 & 0 \\
0.2182 & 0 & 0
\end{bmatrix},
\]

\[
\Delta B_1 = 0.05 \delta(t) \begin{bmatrix}
-0.3571 \\
0 \\
0
\end{bmatrix}.
\]
with $b = \frac{10\pi}{\sigma}$ and $-1 \leq \delta(t) \leq 1$. The uncertainties in the system can be modelled as $\Delta A_1 = \Delta A_2 = \Delta A_{11} = \Delta A_{21} = MF(t)E, \Delta B_1 = M_b F(t)E_{b1}$, and $\Delta B_2 = M_b F(t)E_{b2}$ with $M = [0.255 \ 0.255 \ 0.255]^T, E = [0.1 \ 0.0], M_B = [0.1790 \ 0 \ 0]^T, E_{b1} = 0.05$, and $E_{b2} = 0.15$. According to [3], the fuzzy membership functions are taken as

$$h_1 = \left(1 - \frac{1}{1 + \exp(-3(\theta(t) - 0.5\pi))}\right) \left(1 + \frac{1}{1 + \exp(-3(\theta(t) + 0.5\pi))}\right), \quad h_2 = 1 - h_1.$$  

Then, Theorem 2 in [20] (with $\rho_2 = 0.51, \rho_3 = -0.21, \rho_4 = 4$) offers the maximum allowable value of $\sigma$ is 2.7536. However, using Theorem 2 with $d = 0.5$, it is found that the maximum allowable value of $\sigma$ is 25. In addition, the following table shows the feedback gain matrices for $\sigma = 25$ and the different values of $d$ in order to give the further explanation of Remark 1.

<table>
<thead>
<tr>
<th>$d$</th>
<th>$K_1$</th>
<th>$K_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$[14.8162 \ -26.9367 \ 1.5280]$</td>
<td>$[15.0887 \ -28.3433 \ 1.5617]$</td>
</tr>
<tr>
<td>1</td>
<td>$[10.3130 \ -13.5197 \ 0.5539]$</td>
<td>$[10.6655 \ -14.6551 \ 0.5766]$</td>
</tr>
<tr>
<td>1.5</td>
<td>$[7.6563 \ -6.9265 \ 0.1991]$</td>
<td>$[7.8700 \ -7.4547 \ 0.2047]$</td>
</tr>
<tr>
<td>2</td>
<td>$[6.1808 \ -4.0596 \ 0.0878]$</td>
<td>$[6.3231 \ -4.2347 \ 0.0896]$</td>
</tr>
</tbody>
</table>

The simulation was carried out for an initial condition $\varphi(t) = [5 \ -3 \ 4]$ and uncertainty $F(t) = \sin(2t)$ for $t \in [-25 \ 0]$. Fig. 4 shows the response of the closed-loop system. Fig. 5 shows the curve of control input.

From the simulation results, it can be clearly seen that for the case of unknown and constant delay, our method offers less conservative results in the sense of getting larger allowable time-delay and smaller feedback control gains than the existing delay-dependent approaches.
6. Conclusion

In this paper, we have studied the problem of stabilization for T–S fuzzy systems with time-delay in the sense of delay-dependent stability. The time-delay is constant and unknown. The main contribution is presenting delay-dependent robust stability criteria and stabilization conditions in terms of LMIs. Some examples show that for the case of constant time-delay the proposed delay-dependent methods are less conservative than the existing delay-dependent methods. The disadvantage of the results is that time-delays must be constant. It is challenging to establish a less conservative delay-dependent stability criterion for the case of time-varying delay.

Acknowledgments

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References