Superposition of helical beams by using a Michelson interferometer

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Abstract: Orbital angular momentum (OAM) of a helical beam is of great interests in the high density optical communication due to its infinite number of eigen-states. In this paper, an experimental setup is realized to the information encoding and decoding on the OAM eigen-states. A hologram designed by the iterative method is used to generate the helical beams, and a Michelson interferometer with two Porro prisms is used for the superposition of two helical beams. The experimental results of the collinear superposition of helical beams and their OAM eigen-states detection are presented.

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References and links

1. Introduction

Orbital angular momentum (OAM) of a helical beam is related to the spiral phase distribution of the field [1–4]. The OAM eigen-states, i.e. the quantum number of the helicity, can be any integer ranging from $-\infty$ to $+\infty$, which means a single photon may carry infinite bits of data in theory. This property suggests the potential application of the OAM in the areas of optical communication, information encryption and etc [5,6].

For the information transmission by using the OAM of light beams, it is crucial to encode the information on the OAM eigen-states. The encoding is usually realized by superposing several helical beams coaxially. Two methods are usually used to achieve this goal. One method is to generate the desired transverse distribution of the light beam by modulating the phase and amplitude of the incident beam individually or simultaneously. For example, the superposition of several non-diffractive beams with different OAM charges or helical beams can be realized by the phase- and amplitude-modulating hologram simulated by the spatial light modulator (SLM) [7–9]. The modulation speed is always limited by the refresh rate of the SLM. The other method is realized by controlling the beam splitting, rotation and combination. In Ref [10] a Sagnac interferometer with a polarizing beam splitter was used to separate the diffractive field into two perpendicularly polarized fields behind a stationary grating, and recombine them again. In this paper, we present an experiment with a set of different helical beams distributed on a circle equidistantly. The beam separation and recombination are controlled by a Michelson interferometer with two Porro prisms. Compared with the setup we used in Ref. [10] it is easier to adjust the system and keep the incident and transmitting light beams coincident when rotating the Porro prisms. Finally, we obtain different combinations of two helical beams. The modulation speed of this system is independent on the refresh rate of the SLM.

2. Theoretical analysis

2.1 OAM multiplexing

Wavelength division multiplexing (WDM) is already widely used in fiber communication system. Similarly, if we have several coaxial helical light beams which can be used to represent different information states, the OAM mode multiplexing can be realized. The SLM is a simple method to generate the information carriers, but the modulation speed is often limited by the refresh rate of the SLM. In this paper we realize the OAM multiplexing by rotating the beams to avoid the speed limitation problem.

Assume a set of helical beams with different OAM states distributed on a ring symmetrically. We separate the beam into two beams, and make them rotate relatively to some angle, and recombine them again. Then a set of different helical beam groups is realized. Figure 1 shows this process. Adjusting the rotation angles $\theta_{cw}$ and $\theta_{ccw}$ such that $\theta_{cw} + \theta_{ccw} = n\pi/4$ ($n$ is an arbitrary integer), we obtain different groups of helical beam superpositions. For 8 helical beams with different OAM orders on a circle as plotted in Fig. 1,
36 combinations can be realized. Generally, \( N(N+1)/2 \) combinations could be obtained with \( N \) different helical beams.

![Scheme of OAM multiplexing.](image)

**2.2 Generation of helical beams with OAM**

According to the above analysis it is necessary to generate a kind of helical beam distribution such as helical beams distributed on a circle symmetrically. In fact, many works concerning the generation of helical beams have been studied [2,8–21]. The helical beams can be generated by the intra-cavity mode selecting method or by the extra-cavity mode conversion method [11–17]. For the extra-cavity mode conversion, several methods were investigated, such as the cylindrical-lens mode converters [13,14], the phase plates [15,16] and the diffractive gratings [17]. The cylindrical-lens mode converter has the advantage to generate pure helical modes with high efficiency, but the optical system is relatively large. The phase plate is difficult to be manufactured due to the high machining accuracy needed. The diffractive grating is a more convenient method to generate the helical beams. We can use the computer generated hologram (CGH) method to design the diffractive gratings, and realize them by a spatial light modulator.

In our previous work, we realized the OAM multiplexing by a Sagnac interferometer, but used only four of eight diffractive modes [10]. The efficiency is lower. Here we generate a set of different helical beams distributed on a circle symmetrically, and then all diffractive modes can be used to carry information.

For equal amplitudes the resulting field reads [20],

\[
S(r, \varphi) = \sum_{n=-N}^{N} \exp\{i n \varphi\} \exp\{i k r \rho_n \cos(\varphi - \theta_n)\}
\]

(1)

where the polar coordinates \((\rho_n, \theta_n)\) define the centre of the \(n\)th diffraction order in the Fourier plane. The Eq. (1) describes a helical beam with angular quantum number \(n\) at the position \((\rho_n, \theta_n)\). All these helical beams carry equal power.

What we need to do now is to find a phase function \(a(\Phi)\), which satisfies

\[
S(r, \varphi) \approx \exp\{i a(\Phi)\}
\]

(2)

This process can be realized by the Gerchberg-Saxton iterative method. In our computation, we start with \(a(\Phi) = \arg (S(r,\varphi))\) as the initial value of the phase. Then the iterative process has good convergence. Some experimental results will be given below.

**2.3 OAM detection of helical beams**

There are some methods to detect the OAM of light beams [4,22–24], such as using Mach-Zehnder interferometer to separate different OAM modes, measuring OAM according to the Doppler effect, and so on. To simplify the measurement operation, we use a binary amplitude modulation hologram to detect OAM states of the superposed helical beams.
Fig. 2. Scheme of the detection system.

As shown in Fig. 2, the detecting grating is a binary amplitude grating with a m-dislocation horizontally and a n-dislocation vertically in its center, which is a binary amplitude version of the type of hologram used in Ref. [5]. It is placed in front of the lens which generates the Fourier transform in the focal plane. The CCD camera placed in the back focal plane of the lens is used to monitor the diffraction pattern. When a helical beam with $l = m$ incidents on an amplitude modulated hologram with $m$ dislocations in center, there will appear a bright Gaussian spot in −1 order in the far field. This is the theoretical basis of the detection system.

3. Experimental setup and discussions

According to the analysis of the section 2, an optical system of information transmission based on OAM multiplexing is built. The whole system includes three parts: generation of helical beams, superposition of two helical beams to achieve encoding, and detection of helical beam superposition to realize decoding. The main structure is shown in Fig. 3.

3.1 Generation of helical beams

The helical beams were generated by propagating a $632.8\text{nm}$ TEM$^{00}$ mode He-Ne laser through a spatial light modulator (LC-R 2500). A polarizer is used behind the expander. The LC-R 2500 SLM has a good phase modulation performance when the incident light beam is a linearly polarized beam. We define the position of the polarizer such that the $0^\circ$ position
blocks horizontally polarized light and transmits vertically polarized light. When the polarizer is tuned to the 170° position, an approximate linear relationship between the input gray level and the corresponding phase modulation can be achieved. Behind the SLM a convex lens is used to obtain the Fourier transform. The SLM and the pinhole are on the front focal plane and the back focal plane, respectively. Set \( N = 4 \) and \( N = 6 \) in formula (1), different helical beams distributed symmetrically on a ring are generated. Figure 4 shows the generated helical beams. A bright spot in the center of the ring is observed. The central bright spot may have a contribution of the specular reflection in the front surface of the SLM and a contribution of a remaining zero diffraction order of the hologram generated with the SLM. We can make use of this bright spot to adjust the beam in the following steps.

![Fig. 4. Experimental results of helical beams distributed symmetrically on a ring.](image)

### 3.2 Setup of OAM multiplexing

The separation, rotation and recombination of helical beams are necessary for OAM multiplexing. Firstly a beam splitter prism is used to divide the incident beam into two beams. We note that for two helical beams with OAM order \( m_1 \) and \( n_1 \), the interference pattern is similar to that of another pair with OAM order \( m_2 \) and \( n_2 \), when \( |m_1 - n_1| = |m_2 - n_2| \). This phenomenon is also discussed in Ref. [25]. To avoid interference between superposed beams and make detection easy, we tune the polarization direction ceaselessly. A Michelson interferometer is built as shown in Fig. 3(b) to superpose the two different helical beams polarized in two mutually perpendicular directions.

Again we use the definition of direction in 3.1. The helical light beams generated in section 3.1 are linearly polarized in 170°, so we can tune the fast axis of a \( \lambda/4 \) wave plate to the position of about 125° to transform these beams into circularly polarized light. Then the optical path is separated into two by a birefringent prism. There is a \( \lambda/4 \) wave plate in each arm of the interferometer, and their optical axes form an angle of about 45° angle with the polarization direction of the transmitting beams. Two Porro prisms rotate the beams in the two arms. The phase difference due to the total reflection of the two beams is compensated by a piece of birefringent crystal coaxially with the Porro prism [26]. When the center of the helical beam ring hits the edge of the prism the ring rotates with the Porro prism. Rotating the Porro prism by \( \theta \), the helical beams ring rotates 2\( \theta \) with the same rotation axis. The beams from two arms recombine after being reflected as Fig. 1 shown. One group can be selected using a pinhole.

In the experiment, Porro prism 1 is rotated clockwise and Porro prism 2 counterclockwise. Figure 5 shows the general far field pattern when rotating these two Porro prisms. We also notice that no interference pattern appear in Fig. 6 because the two polarized parts polarize perpendicularly to each other. It's clear that there will be many angular positions realizing superposition of two beams and an off-axial pinhole would be used to filter one helical beam superposition group. We could get all of the 36 different superposition groups by rotating Porro prism 1 and Porro prism 2.
3.3 Detection of helical beam groups

An amplitude modulation grating with 1-dislocation in horizontal direction and 3-dislocations in vertical direction in its center was chosen to detect the OAM of the filtered helical beams groups. This is sufficient for the detection of the helical beam superposition groups based on Fig. 4(a).

Figure 6 gives some experimental results of detecting different superposed helical beam groups by a binary amplitude grating. Porro prism 1 and 2 are rotated to obtain the superposition of two beams. For example, Fig. 6(a) is obtained when $\theta_{cw} = \pi/8$ and $\theta_{ccw} = 3\pi/8$. The beam reflected by Porro prism 1 is rotated by an angle of $\pi/4$ clockwise, while the beam reflected by Porro prism 2 is rotated by of $3\pi/4$ counter-clockwise. That means a helical beam with azimuthal number +4 is coaxial with other helical beam with azimuthal number −4, and can be selected by a pinhole. When such superposed beams irradiate the measuring grating in Fig. 6, a bright spot will appear on the position of −4 and +4 diffractive order, respectively. This holds for all other cases. According to analysis, the OAM states of the
coaxial superposed beams could be identified and labeled in Fig. 6. Only 9 of the 36 groups experimental results are given, and the others are not shown here.

4. Conclusion

In this paper we describe the generation of a set of helical beams distributed symmetrically on a ring, and realized the different superposed groups of two helical beams using a Michelson interferometer with two Porro prisms. The OAM states of the helical beams are detected in the experiments. The optical system can increase the information capacity of light beams considerably. This work is of importance for optical communication based on the OAM states of laser beams.

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