Investigating upper bounds on lifetime for target tracking sensor networks

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Abstract: The distinctive characteristic of target tracking sensor networks is that the delay of data transmission is constrained, which poses a difficult problem for predicting the application lifetime for such sensor networks. In this paper, we first map the delay constraints to the hop bound in routing. By analysing the energy consumption in recurring bounded hop routing, we establish the relationship between individual sensors and the whole sensor network. On the basis of this relationship, we propose a novel model to formally define the lifetime of target tracking sensor networks. The model not only implies the best routing strategy, but also exposes the dependence of lifetime on factors such as hop bound, radio transmission range, sensing range and target behaviour. Finally, we present the results of extensive simulation to investigate the influence of some adjustable parameters in engineering on the lifetime.

Keywords: wireless sensor network; target tracking; lifetime; routing.


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1 Introduction

Rapid commoditisation and increasing integration of microsensors, digital signal processing and low-range radio electronics on a single node has led to the idea of distributed, wireless networks that have the potential to collect data more cost-effectively, autonomously and robustly compared to a few macrosensors. Applications of such massively distributed sensor networks include seismic, acoustic, medical and intelligence data gathering and climate, equipment monitoring, etc.

Due to the wide range of application areas, sensor networks present a variety of network operation models for data delivery and processing. Three models are identified in Tilak et al. (2002), namely the continuous, event-driven and user-initiated. In this paper, we focus on the second type, which represents applications like target tracking. In this type of application, the user is only interested in the occurrence of a certain event or a set of events like appearance or tracks of the target.

Since the integrated sensor nodes have highly compact form factors and are wireless, they are highly energy constrained. Furthermore, replenishing energy via replacing batteries on up to tens of thousands of nodes (in possibly harsh terrain) is infeasible. Hence, it is well accepted that one of the key challenges for such target tracking sensor networks is conserving energy so as to maximise their post-deployment active lifetime. Most target tracking applications are interactive, delay intolerant and mission critical. It means that the
application needs to detect the specific events and accordingly takes an appropriate action as quickly as possible. Therefore, bounding the sensors-to-base-station data transmission delay so as to satisfy the Quality of Tracking (QoT) requirements is another key challenge for target tracking sensor networks.

How to extend the lifetime of sensor network has been a topic of considerable interest in the research field of sensor networks. Many efforts have been made to achieve this goal by using energy-efficient protocols such as Al-Karaki and Kamal (2004). On the other hand, studies on Quality of Service (QoS) support in sensor networks are booming. Some representative work that adequately addresses the real-time requirements is in He et al. (2003).

In this paper, our key objective is neither proposing new energy-aware routing heuristics nor new QoS-based routing protocols. Instead, it is to explore the fundamental limits of the target tracking lifetimes that these strategies strive to increase. Our motivation for doing so is several-fold. First, bounds on achievable lifetime of sensor networks that both consider the energy constraint of single node and the end-to-end delay constraint of data transmission allow sensor system designers to be more aware of the capability of sensor networks to provide the services it was designed for independent of the protocols and algorithms used. Second, in order to guarantee that the proposed bounds are tight or near tight, we analyse the energy consumption step by step in data generating and delivery. This analysis gives an insight into how to build optimal routing paths to prolong lifetime. Third, in bounding lifetime, we expose its dependence on target behaviour, number of sensor nodes, available initial energy, sensing range and radio energy parameters. This allows us to see what factors have most impact on lifetime and consequently where engineering effort is best expended.

The rest of the paper is organised as follows. In the next section, we summarise the related work. In Section 3, we state the assumptions we make on the system model, define what we mean by network lifetime in sensor networks. Then, we summarise the results of lifetime upper bound in wireless sensor network in Section 4. In Section 5, we present simulation results to investigate the influence of some key factors on the lifetime. Finally, we conclude the paper in Section 6.

2 Related work

In this section, we present related work that deals with lifetime bounds of sensor networks. To the best of our knowledge, none of these approaches has explicitly considered the real-time requirements in their models. In several previous works, the lifetime of the sensor network is defined as the time for the first node to run out of power such as in Heinzelman et al. (2000) or a certain percentage of network nodes to run out of power as in Ephemrides (2002). We think that these definitions of the lifetime of the sensor network are too pessimistic as when only some nodes fail the others can still provide the whole sensor network appropriate functionality.

In the work of Giridhar and Kumar (2005), Bhadwaj et al. (2001), Bhadwaj and Chandrakasan (2002), Blough and Santi (2002), Alonso et al., (2004) and Sha and Shi (2005), the lifetime of the sensor network is defined as the time when the sensor network first loses connectivity. The rationale of their definition is based on the data delivery functionality of the sensor network. Giridhar and Kumar (2005) call it functional lifetime. Bhadwaj et al. (2001) first formally defined this functional lifetime bound of a sensor network for basic data gathering scenarios. In later work, the authors (Bhadwaj and Chandrakasan, 2002) extend their analysis by including data aggregation and network topology. They explore the fundamental limits of the lifetime, and map the lifetime bounding to a linear programming problem. But they just give out an upper bound in theory, and do not provide any practical way to achieve the bound. Blough and Santi (2002) study the upper bound of the lifetime for cell-based energy conservation techniques, which is restricted to the Geographic Adaptive Fidelity (GAF) scheme. The lifetime is determined by checking the connectivity of a graph. Alonso et al. (2004) and Sha and Shi (2005) formally define the lifetime by considering the relationship between individual sensors and the whole sensor network. They determine the network lifetime by calculating the lifetime of the key nodes that will lead to the network connection broken.

However, a definition of network lifetime expressed solely in terms of the ‘capability of communication’ is not sufficient either. Blough and Santi (2002) advocate the requirement for connectivity must be complemented with the requirement for coverage. They focus on the sensing coverage functionality of the sensor network. We call it covering lifetime. Zhang and Hou (2004) have proved if the sensor’s transmission range is at least twice the sensing range, complete coverage of a convex region implies connectivity. That is, as long as the set of working nodes completely covers the monitored region, the network is connected. Based on this observation, they provide the upper bounds of $a$-lifetime (lifetime when only $a$-portion of the region is required to be covered at any time), and device a centralised algorithm that maximises the $a$-lifetime of sensor networks. Covering lifetime model is the beginning of considering QoS requirement in lifetime modelling, and should be extended by considering other QoS requirements. Blough and Santi outline the principle that a good definition of network lifetime should refer to the capability of the network to provide the services it was designed for, and hence depends on the application scenario. In this paper, we examine the scenario in which the sensors that detect the target have to transmit their readings to a base station on time, and try to explore how long the sensor network can provide this real-time tracking service.

3 Preliminary

To facilitate the derivation, in this section we state the assumptions we make on the system model, and define the lifetime for target tracking sensor networks.
3.1 Target tracking sensor network model

Consider a static network of \( N \) homogeneous sensor nodes and a base station node distributed over a big area \( A \) with uniform density \( \rho \). Each sensor has a battery with finite, unreplenishable energy \( E \), whereas the base station has an unlimited amount of energy available to it. The sensor network attempts to sense and track objects as they move through the area. When a sensor node detects the target, it needs to transmit its sensing data to the base station node. We assume that each sensor generates one data packet per time unit. For simplicity, we refer to each time unit as a round.

For the sensor network is deployed in a big region, it needs to use multi-hop forwarding. The end-to-end data transmission delay is the sum of delays experienced by each hop from the data source node to the base station node. The delay at each intermediate node has two components: a fixed delay which includes the transmission at sender node and the propagation over the link to the next node, and a variable delay which includes the processing and queuing at sender node. For all sensor nodes are homogeneous and have the same behaviours (run the same programs), we assume the delay per hop is the same along the path (denoted \( \text{HopDelay} \)). Below, we use this assumption to map the end-to-end delay constraints to the bounds on path length.

3.2 Target behaviour model

For the dominant traffic in the sensor network is completely dependent on the target behaviour, it is necessary to specify how the target resides in \( A \). Barring few exceptions, the target behaviour is not known in a deterministic sense at the time the network is deployed. Rather, one must make do with stochastic knowledge of target behaviour. In this paper, we use a simple but effective stochastic model – the spatial probability density function of a target (denoted by \( f(x,y) \)) with the usual properties (Bhardwaj et al., 2001).

\[
\Pr(\text{target } \in D) = \iiint_D f(x_i, y_i)\,dx_i\,dy_i
\]

\[
\iiint_D f(x, y)\,dx\,dy = 1
\]

Next, it is important to note that sources have finite regions of observability. We assume circularly observable sources with a radius of observation equal to \( d_s \). This implies that only live nodes less than \( d_s \) away can observe the source.

3.3 Energy consumption model

Every node has a sensor, analog pre-conditioning and data conversion circuitry, digital signal processing and a radio link. Since the dominant energy consumer is the radio transceiver, we only consider the energy consumption in the process of data communication that involves transmission, reception, and being idle. In most of the past research, wireless transceivers are assumed to consume power only when transmitting packets, and energy is thus consumed on a per-packet basis. However, wireless transceivers also consume energy during idle periods because they have to be powered to detect if there are incoming packets. In this paper, we include this energy consumption in our model and define the parameters as follows:

- \( E_{\text{idle}} \): energy used in idle listening during one round
- \( E_{\text{rx}} \): energy required for receiving a \( k \)-bit packet
- \( E_{\text{tx}} \): energy required for transmitting a \( k \)-bit packet.

Assuming a \( 1/\epsilon \)\( ^2 \) path loss, \( E_{\text{tx}} \) and \( E_{\text{rx}} \) take the following form:

\[
E_{\text{rx}} = e_{\text{elec}} \times k
\]

\[
E_{\text{tx}} = e_{\text{elec}} \times k + e_{\text{amp}} \times \epsilon \times k
\]

Here \( e_{\text{elec}} \) is the energy/bit consumed by the transmitter or receiver electronics (including energy costs of imperfect duty cycling due to finite startup time), \( e_{\text{amp}} \) accounts for energy dissipated in the transmit op-amp (including op-amp inefficiencies).

There is no elegant form like equation (2) to model \( E_{\text{idle}} \). Researchers have shown that in some cases, \( E_{\text{idle}} \) is comparable to \( E_{\text{tx}} \) and \( E_{\text{rx}} \). Specifically, wireless sensor networks are presumed to be densely deployed, and this has two implications. On the one hand, pairwise distance between sensor nodes is small, and thus packet transmission between sensor nodes consumes less energy. On the other hand, each sensor node covers more sensor nodes in its transmission range, and thus more energy will be consumed by overhearing. We define a ratio \( ER = E_{\text{idle}} : E_{\text{rx}} : E_{\text{tx}} \), and set the value \( ER = 1:2:2.5 \) as observed in Ergen and Varaiya (2005).

To simplify the quality lifetime modelling, we further assume that all nodes transmit at the same constant power. So \( E_{\text{tx}} \), \( E_{\text{rx}} \) and \( E_{\text{idle}} \) have the same value for each node. Based on the first-order radio model described in Heinzelman et al. (2000), we have \( e_{\text{elec}} = 50 \text{ nJ}/\text{bit} \), \( e_{\text{amp}} = 100 \text{ pJ}/\text{bit}/\text{m}^2 \).

3.4 Quality lifetime definition

In a target tracking scenario, a sensor network can be in one of the following states:

1. Target present and network sensing while satisfying the quality of tracking requirement (end-to-end delay constraint). This state is termed ‘active’.
2. Target present and network sensing but not satisfying the quality of tracking requirement. This state is termed ‘quality failure’.
3. End-to-end connectivity of the network is broken, no data can be sent to the base station node. This state is termed ‘connectivity broken’.
4. In this paper, lifetime is defined as the cumulative active time of the network until the first quality failure. We call it Quality Lifetime (QL).
4 Quality lifetime model

A recurring theme in active time of target tracking networks is the problem of establishing routes between data sources and the base station node. For each round \( t, 1 \leq t \leq T \), every data source sends a packet of length \( k \) to the base station. Formally, a routing is a vector \( y = (y_i)_{t \leq i \leq T} \), where \( y_i \) represents the total number of packets that are sent by intermediate node \( i \) during the \( t \)th round. Observe that we can think of the routing \( y \) as being a sequence \( (y_i)_{1 \leq i \leq T} \), where \( y_t \) is the routing used during the \( t \)th round. The only restriction we place on routings is that they should satisfy the end-to-end delay constraints. That is, according to the end-to-end delay constraint (denoted \( \Gamma \)), the bound of the hop number of routings is \( \max \text{HopDelay} = \Gamma / \text{MaxLen} \). So, not all of the intermediate nodes between data sources and the base station are eligible to participate in routing. We call the nodes that can construct a routing shorter than \( \max \text{HopDelay} \) the ‘eligible nodes’.

In this section, we propose a novel model to formally define the quality lifetime of a target tracking sensor network based on energy by considering the relationship between eligible nodes and the whole sensor network.

4.1 Sensor nodes classification

The hypothesis that all nodes transmit with the same power implies that all nodes have the same radio transmission range \( h \). Based on this range, we partition the set of all sensor nodes \( V \) into subsets \( S_0, S_1, \ldots, S_n \), satisfying \( V = S_0 \cup S_1 \cup \cdots \cup S_n \), \( S_i \cap S_j = \emptyset \) for all \( i \neq j \) and no \( S_i \) is empty. \( S_i \) is the set of nodes that can be reached from the base station node \( B \) in \( i \) hops \( (S_0 = \{B\}) \), but not less than \( i \) hops. We call \( S_i \) the sphere of radius \( i \) around \( B \). \( s_i = |S_i| \) is the total number of the sensor nodes in \( S_i \). The goal of this division is to classify the nodes in the network by their capability in data delivery. In delay bounded routing, the data delivery capability is measured by the distance from the node to the base station node. As we assume the delay per hop is the same along the path, the nodes in the same sphere can transmit data to the base station node with the same delay, so they may have the same energy consumption model in routing. In our model, we simplify the energy consumption analysis of the whole network by exploring the energy consumption of the nodes in each sphere.

4.2 Eligible nodes in each sphere

After classifying the sensor nodes into spheres, we show how to calculate the number of eligible nodes in each sphere, which is very important to calculate the energy consumed by the whole network. According to the end-to-end delay constraints, the distance between data sources and the eligible nodes in sphere \( S_i \) should be less than \( \max \text{HopDelay} - i \).

We set \( i = 2 \), \( \max \text{HopDelay} = 5 \) as an example. The eligible nodes in \( S_i \) must reside in the overlapping area (denoted by \( S_{ei} \)), as shown in Figure 1, where the number of the eligible nodes is \( n_i = \rho \cdot S_{ei} \).

In this subsection we show how to calculate the overlapping area of two circles, based on which we will show how to calculate the eligible nodes.

**Figure 1** Overlapping area

4.2.1 Overlapping area calculation

Suppose the radii of two circles (see Figure 2) are \( R \) and \( r \), respectively, where \( R \geq r \). Let \( x \) denote the distance between two centres of the circles, where \( x \geq 0 \).

![Overlapping area of two circles](image)

We calculate the overlapping area \( S_{Rr}(x) \) using integration, as shown in equation (3).

\[
S_{Rr}(x) = \begin{cases} \pi \cdot r^2 & 0 \leq x \leq R - r \\
\frac{R^2 \arccos \left( \frac{x^2 - r^2 + R^2}{2xr} \right)}{2} + \frac{1}{2} \sqrt{4x^2 R^2 - \left( x^2 - r^2 + R^2 \right)^2} & R - r \leq x \leq R + r \\
0 & R + r \leq x \end{cases}
\]  

(3)

To simplify \( S_{Rr}(x) \), we use a piece-wise linear function \( S_{Rr}(x) \) to approximate \( S_{Rr}(x) \).
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\[ \tilde{S}_b(x) = \left\{ \begin{array}{ll} \pi \cdot r^2 & 0 \leq x \leq R - r \\ S_b(R) + S_b'(x)(R-x) & R-r < x < R + r \\ 0 & R + r \leq x \end{array} \right. \]  

(4)

\[ S_b(R) = R^2 \arccos \frac{2R^2 - r^2}{2R^2} + r^2 \arccos \frac{r}{2R} - \frac{1}{2} \sqrt{4R^2 - r^2} \]  

(5)

Then, the mean overlapping area can be calculated as:

\[ E(S_{el}) = \int_0^{M_b} S_{el}(x)f(x)dx = \int_0^{M_b} (S_{R1/2}(x) - S_{R2/2}(x))f(x)dx \]  

(8)

The number of the eligible nodes is \( n_i = \rho \cdot E(S_{el}) \). For simplification, according to equation (4), we get

\[ n_i \approx \rho \int_0^{M_b} (\tilde{S}_{R1/2}(x) - \tilde{S}_{R2/2}(x))f(x)dx \]  

(9)

4.3 Energy consumption during one round

In each round, we assume \( n_s \) nodes can detect a target, where \( n_s = \pi \cdot d_s^2 \cdot \rho \). For most of recently developed sensors, the transmission range is at least twice the sensing range. So, it is most likely the sensors that detect the target are all located in the same sphere. Further we do not consider data aggregation in our model. This means the sensing data is transmitted unchanged to the base station.

Corresponding to the sphere \( S_i \), we introduce balls of radius \( i \) denoted \( B_i \), with \( B_i = S_i \cup \cdots \cup S_j \) and cirques outside \( B_i \) denoted \( O_i \), with \( O_i = V - B_i \). Below, we analyze the energy consumption of sensor nodes in sphere \( S_i \) in the following three cases:

1. We assume the target has the probability \( P_{i-1} \) to be in \( O_i \), \( P_{i-1} = \int_{O_i} f(x,y)dxdy \). When the target is in \( O_i \), the eligible nodes in sphere \( S_i \) are responsible for relaying packets from sphere \( S_{i-1} \) to the base station. The nodes that do not participate in routing will be idle. We calculate the energy consumption for the node in \( S_i \) in this case (Case 1), as

\[ m_{i-1} = \frac{P_{i-1} \cdot n_i}{n_s} (E_{ex} + E_{idle}) + \frac{P_{i-1} \cdot (n_i - n_s)}{n_s} E_{idle} \]  

(10)

Here, \( P_{i-1} \cdot n_s \) is the total number of packets that the set of nodes in sphere \( S_i \) receive in each round; \( n_i \) is the number of eligible nodes of \( S_i \), \( P_{i-1} \cdot n_i/n_s \) is the average number of packets relayed by each eligible node of \( S_i \). Except for \( n_i \) nodes participate in routing, the rest \( n_i - n_s \) nodes will stay in idle state, thus

\[ \left\{ P_{i-1} \cdot (n_i - n_s)/n_s \right\} E_{idle} \]  

is the average energy consumed by each eligible node in over hearing.

2. We assume the target has the probability \( P_{i-1} \) to be in \( S_i \), \( P_s = \int_{S_i} f(x,y)dxdy \). When the target is in \( S_i \), sensor nodes that detect the target act as data sources and send packets to \( S_{i-1} \), other nodes stay at idle. The energy consumption in this case (Case 2) is shown in equation (11).

\[ m_s = \frac{P_s \cdot n_s}{s_i} E_{ex} + \frac{P_s \cdot (s_i - n_s)}{s_i} E_{idle} \]  

(11)

Here, \( P_s \cdot n_s \) is the total number of packets generated in each round. Besides \( n_s \) data sources, all other \( s_i - n_s \) nodes consume energy due to over hearing.

3. We assume the target has the probability \( P_{i+1} \) to be in \( B_{i+1} \), \( P_{i+1} = \int_{B_{i+1}} f(x,y)dxdy \). When the target is in \( B_{i+1} \), all nodes in sphere \( S_i \) will be idle. The energy consumption in this case (Case 3) is shown in equation (12).

\[ m_{i+1} = P_{i+1} \cdot E_{idle} \]  

(12)

We integrate the energy consumption of the above three cases, and define the energy consumption model for the node in \( S_i \) during one round as shown in the following equation.
\[ m_i = m_i^0 + m_i^m + m_{m_i} \]  

Here \( m_i \) measures the energy cost incurred by a node \( S_i \) when it transmits, receives and stays idle during target tracking.

### 4.4 Upper bounds on quality lifetime

Based on the energy consumption model of each sphere, we define the quality lifetime of the target tracking sensor network as shown in the following equation.

\[
QL = \frac{E}{\max \{m_1, m_2, \ldots, m_n\}}
\]  

(14)

In the above equation, \( E \) is the initial energy level of each node; \( \max \{m_1, m_2, \ldots, m_n\} \) is the most energy consumption of all nodes during one round. This equation establishes a relationship between the energy consumption of a signal node and the quality lifetime of the whole network.

Theorem: \( QL \) provides an upper bound on the quality lifetime of target tracking sensor networks.

**Proof:** For each sphere \( S_i \), Cases 1–3 cover all possible energy consumption of the node in \( S_i \) for data communication when tracking targets. If the nodes in sphere \( S_i \) have the \( \max \{m_1, m_2, \ldots, m_n\} \), i.e. the nodes in sphere \( S_i \) consume more energy than the nodes in other spheres during one round, their energy will be first drained out in the network. We call \( S_i \) the bottleneck sphere. Packets from \( O \) can only reach \( S_i \) by going through \( S_i \). When the eligible nodes in sphere \( S_i \) failed resulting from the deplete of energy, the sensing data outside \( S_i \) will not reach the base station node on time, which causes quality failure. Equations (10)–(12) give the average energy consumed by each eligible node during one round in theory. In this model, control overhead and energy waste in data communication, e.g. energy spent in the medium access control, are neglected; all eligible nodes in the bottleneck sphere run out of energy during the same round. So \( QL \) provides an upper bound on the quality lifetime of target tracking sensor networks.

### 4.5 Discussion

We have recently learnt (Alonso et al., 2004; Sha and Shi, 2005) that some researchers have proposed to simplify the lifetime modelling by establishing the relationship between the individual sensor and the whole network. Inspired by this idea, we classify the nodes in the network by their data delivery capability and bound the quality lifetime of target tracking sensor networks by the lifetime of eligible nodes in the bottleneck sphere. The novelty of our model is that it is based on the sensor networks operating in event driven model, and explicitly considers the end-to-end delay constraint and the overhearing energy consumption in data communication.

The model is valuable for three reasons. First, it provides a practical way to forecast how long the sensor network can provide real-time tracking services before deployment. Second, it exposes that the criteria for choosing eligible nodes to participate in routing is decisive in maximising the quality lifetime, and the best the routing algorithm can do is to balance the traffic evenly between the eligible nodes in the bottleneck sphere. Third, it shows what factors have the most impact on quality lifetime. According to equations (10)–(13), quality lifetime is determined by the parameters such as \( n_s, n_A, s_i, P_D, P_n, P_{m_i} \), in which \( n_s \) correlates with hop bound \( MaxLen \), network density \( \rho \) and radio transmission range \( h \); \( n_A \) correlates with network density \( \rho \) and sensing range \( d \); \( s_i \) correlates with network density \( \rho \) and radio transmission range \( h \); \( P_D, P_n, P_{m_i} \) correlate with radio transmission range \( h \) and spatial probability distribution function \( f(x, y) \). As the network density occurs both in the numerator and denominator of equations (12) and (13), the key factors that should be considered for the design of an effective sensor network include hop bound, radio transmission range, sensing range and target behaviour within the region.

### 5 Simulation results

In the above section, we have discussed the factors have the most impact on quality lifetime. Among these factors, hop bound, radio transmission range and sensing range could be adjusted in engineering.

In this section we present the results of extensive simulation. The goal of these simulations was to put a quantitative analysis on the influence of these adjustable parameters on the quality lifetime \( QL \).

The simulator distributes \( n \) nodes in a monitored area \( R = L^2 \), according to the uniform distribution. The base station node resides at the centre of the area; its coordinate is \((0,0)\).

For the target behaviour is not adjustable, to simplify the simulation, we discuss two kinds of general distributions: uniform and normal. When the target is distributed uniformly at random, we assume \( x \) and \( y \) are two independent random variables and each is uniformly distributed at random in \([-\frac{L}{2}, \frac{L}{2}]\). So we get

\[
f(x, y) = \frac{1}{L^2} \quad \frac{L}{2} \leq x \leq \frac{L}{2}, \quad \frac{L}{2} \leq y \leq \frac{L}{2}
\]  

(15)

According to equation (15) we get:

\[
\begin{align*}
F(x) &= \begin{cases} 
\frac{\pi x^2}{L^2} & \frac{L}{2} \leq x < \frac{\sqrt{2}}{2} L \\
1 & L \leq x
\end{cases} 
\end{align*}
\]  

(16)
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\[
f(x) = \begin{cases} 
\frac{2\pi x}{L^2} & x \leq \frac{L}{2} \\
\frac{L}{2} < x < \frac{\sqrt{2}}{2}L \\
0 & L \leq x 
\end{cases}
\] (17)

When the target is distributed normally at random, we also assume \(x_t, y_t\) are two independent random variables and each is normally distributed with parameters \((0, \sigma)\). So we get

\[
f(x_t, y_t) = \frac{1}{2\pi\sigma^2} e^{-\left(\frac{x_t^2+y_t^2}{2\sigma^2}\right)} - \infty < x_t, y_t < \infty
\] (18)

In order to make the target appear in the monitored area with probability greater than 0.9973*0.9973, we set \(\sigma = L/6\).

\[
F(x) = \int_{x^2+y^2 \leq x^2 \sigma^2} f(x, y) \, dx \, dy,
\]

\[
= \int_{x^2+y^2 \leq x^2 \sigma^2} \frac{1}{2\pi\sigma^2} e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)} \, dx \, dy = 1 - e^{\left(\frac{1}{2\sigma^2}\right)} 0 \leq x
\] (19)

\[
f(x) = \frac{x}{\sigma} e^{-\left(x^2/2\sigma^2\right)} 0 \leq x
\] (20)

Note that \(f(x)\) is a Rayleigh distribution.

Below, we evaluate the actual quality lifetime for both target distributions. We denote the Quality Lifetime when target is distributed uniformly at random as \(UQL\) and denote the Quality Lifetime when target is distributed normally at random as \(NQL\). Other parameters are set as follows: \(\rho = 0.05, \ L = 500, \sigma = L/6, \ E_{idle} = 1, \ E_{rx} = 2, \ E_{tx} = 2.5\) and \(E = 1000\).

We performed separate sets of simulations to investigate each of the adjustable parameters. Simulation results, all average over 200 runs, are reported in the following sub-sections.

5.1 The energy consumption in each sphere

The first set of simulations was aimed at exposing the bottleneck sphere in the uniformly distributed target tracking sensor networks. This makes it easy to explain our following simulation results. Parameters are chosen as the transmission range \(h = 30\), the hop bound \(M = 8\) and the sensing range \(d = 5\), which obey the following rule: the transmission range is at least twice the sensing range.

Simulation results, which are reported in Figure 4, show that the energy consumed in inner spheres is higher than that in outer spheres. This happens because all data packets must pass through the inner spheres before they reach the base station node. Sphere 1 is the bottleneck sphere, for it has the least node number and the heaviest workload among all spheres. Furthermore, the energy consumption in inner spheres for tracking a uniformly distributed target is less than that for tracking a normally distributed target, and the result is reverse in outer spheres. The reason is that the frequency of the uniformly distributed target presents in inner sphere is less than that of the normally distributed target.

![Figure 4](online version for colours)

5.2 Sensing range vs. quality lifetime

In this set of simulations, we have investigated the effect of a different value of the sensing range on quality lifetime. We keep the transmission range \(h = 30\), the hop bound \(M = 8\), and vary the sensing range \(d\) from 2 to 15. The result is shown in Figure 5. For both target behaviour distribution, the quality lifetime decreases linearly as the sensing range increases. This happens because the number of data source that can detect the target increases as the sensing range increases, which causes enhanced traffic in the network. The additional data packets cost more transmitting energy and receiving energy in each sphere. Furthermore, the \(UQL\) is greater than \(NQL\) at the same sensing range point. This happens because the quality lifetime of the network is determined by the lifetime of the bottleneck Sphere 1. As discussed in the above subsection, the energy consumption in this sphere for tracking a normally distributed target is higher than that for tracking a uniformly distributed target.

![Figure 5](online version for colours)

5.3 Hop bound vs. quality lifetime

The hop bound reflects the end-to-end delay constraint and is application specific. The goal of this set of simulation was to investigate how the users prolong the network lifetime by adjusting their real-time requirements. In this situation, we keep the transmission range \(h = 30\), the sensing range
$d=5$, and vary the hop bound $M$ from 8 to 16. As shown in Figure 6, both $UQL$ and $NQL$ increase as the increases of $M$, and reach the maximum value at $M=11$. This happens because Sphere 1 is the bottleneck sphere, and when $M=11$ all nodes in this sphere are eligible nodes. The results tell us properly relaxing the delay constraints is an efficient method to achieve a longer network lifetime.

**Figure 6** Quality lifetime vs. $M$ (see online version for colours)

5.4 Transmission range vs. quality lifetime

Adjusting the radio transmission range is a popular method to control the energy consumption in each node. Most commercial radio chips like chipcon cc1000 provide interfaces for users to configure the desired transmission range. This last set of simulations was to investigate how the transmission range $h$ influences the quality lifetime. We keep the sensing range $d=5$, the hop bound $M=15$, and vary the transmission range $h$ from 10 to 30. The result is shown in Figure 7. For both target behaviour distribution, the quality lifetime increases while the transmission range increases. This happens because the increase of the transmission range enlarges the bottleneck sphere.

The total energy consumed in each sphere changes slightly. Furthermore, when the transmission range reaches 100, the difference between $NQL$ and $UQL$ is very small.

**Figure 7** Quality lifetime vs. transmission range ($h$) (see online version for colours)

6 Conclusions

The key challenge in networks of energy constrained wireless integrated sensor nodes is maximising network lifetime. In this paper, we derived fundamental upper bounds on the lifetime of target tracking sensor networks. The main idea is to model the bound by establishing a relationship between the lifetime of the whole network and that of the eligible nodes in the bottleneck sphere.

The model exposes the key factors that have the greatest impact on lifetime, such as hop bound, radio transmission range, sensing range and spatial probability distribution function of the target. This theoretical result has been validated by means of extensive simulations, which have complemented the theoretical with a quantitative analysis. In particular, the results of our simulations have shown that the relaxed hop bounds, the longer radio transmission range and the shorter sensing range will extend the lifetime. The energy consumption analysis in modelling also gives an insight into how to build optimal routing paths to prolong lifetime.

While the model explicitly considers the end-to-end delay constraint and the overhearing energy consumption in data communication, other practical concerns, chief amongst them the data aggregation in network, remains to be incorporated. We hope that the work reported here will provide a starting point in constructing the ultimate bounds on the lifetime of target tracking sensor networks.

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