Focusing properties of the double-vortex beams through a high numerical-aperture objective

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A B S T R A C T
We study the focusing properties of the double-vortex beams through a high numerical-aperture objective based on the vectorial Debye theory. We investigate the double-vortex beams composing of two rings, and each ring carries different orbital angular momentum (OAM). It is shown that the total intensity distributions in the focal region are of double-ring structure, and by adjusting certain parameters, the shape of double-ring can be changed. The tight focusing of the double-vortex beams may find applications in micro-particle trapping, manipulation, material processing, etc.

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1. Introduction

It is well known that when a light beam is focused by a high numerical aperture (NA) objective lens, we can achieve an extremely small spot near the focus. Moreover, the higher the NA is, the smaller the focused spot is [1–5]. For its unique properties, tightly focused light beams have wide potential applications in optical data storage, microscopy, material processing, micro-particle trapping manipulation, etc. [6–9]. On the other hand, a great deal of attention has been given to the research of the vortex beam since 1992. The vortex beam, also known as the helical beam, has a continuous spiral phase wave front, which carries an orbital angular momentum (OAM) [10,11]. Owing to have wide applications in quantum information, the optical trapping and rotating of micro-particles, it is important to study on the tight focusing of the vortex beams through a high NA objective [12–15]. In recent years there has been much research on tight focusing of different kinds of vortex beams, such as linearly polarized, circularly polarized, elliptical polarized vortex beams [16–18]. However, to the best of our knowledge, there are no papers studying the focusing of the double-vortex beams through a high NA objective.

In this paper, based on the vectorial Debye theory [19], we study the tight focusing of the double-vortex beams in detail. We find that the total intensity distribution in the focal region is also of double-ring structure, and its shape can be changed by adjusting relative parameters. In addition, the double-vortex beams can carry two different topological charges. The results will be important in the micro-particle trapping and manipulation.

2. Theory

The Laguerre–Gauss beam is a typical example of vortex beams. In this paper, we take the linear polarized Laguerre–Gauss beam \( LG_m^p \) as the double-vortex model. Considering the mode index \( p = 0 \), the transversal amplitude distribution of a completely coherent Laguerre–Gaussian beam \( LG_m^0 \) can be expressed as [20]

\[
E_m(r,\phi) = E_0 \left( \frac{\sqrt{2r}}{w_0} \right)^{|m|} \exp\left( -\frac{r^2}{w_0^2} \right) \exp(i m \phi), \quad j = x, y, \tag{1}
\]

where \( r \) is the radial coordinate around the optical axis, \( \phi \) is the polar coordinate in the plane perpendicular to the beam axis, \( E_0 \) is the constant amplitude, \( w_0 \) is the beam size and \( m \) is the topological charge. For a commercial objective, the sine condition \( r = f \sin \theta \) is usually obeyed in design processes, where \( f \) is the focal length of the high NA objective, and \( \sin \theta \) is the NA angle. Then Eq. (1) can be transformed to [19]

\[
A_m(\theta,\phi) = A_m(\theta) \exp(i m \phi) = E_0 \left( \frac{\sqrt{2f \sin \theta}}{w_0} \right)^{|m|} \exp\left( -\frac{f^2 \sin^2 \theta}{w_0^2} \right) \exp(i m \phi) \quad j = x, y, \tag{2}
\]

where \( A_m(\theta) \) is called the apodization function.

Here we consider the case that the double-vortex beams are generated by coherent superimposition of two orthogonal linearly polarized vortex beams with different topological charges \( m_1 \) and \( m_2 (m_1 < m_2) \), respectively. The double-vortex beams can be denoted as

\[
\hat{E}(r,\phi,z) = \hat{E}_1(r,\phi,z) \cdot \mathbf{e}_x + \hat{E}_2(r,\phi,z) \cdot \mathbf{e}_y, \tag{3}
\]

where \( \hat{E}_1 \) and \( \hat{E}_2 \) are two orthogonal linearly polarized vortex beams, \( \mathbf{e}_x \) and \( \mathbf{e}_y \) are unit vector in the \( x \) and \( y \) directions, respectively.
According to the vectorial Debye theory, when an x-polarized vortex beam is focused through a high NA objective, the electric wave in the focal region can be expressed as \[1,21\]

\[
E_t(r,\phi,z) = \begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix} = f^m \begin{bmatrix}
-iA[l_0 + 0.5i(l_{m+2}e^{2i\phi} + l_{m-2}e^{-2i\phi})] \\
-iA[0.5i(l_{m+2}e^{2i\phi} - l_{m-2}e^{-2i\phi})] \\
-iA[l_{m+1}e^{i\phi} - l_{m-1}e^{-i\phi}]
\end{bmatrix} e^{im\phi},
\]

(4)

where \(r, \phi \) and \(z \) are the cylindrical coordinates of an observation point, \(E_x, E_y \) and \(E_z \) are, respectively, \(x-, y- \) and \(z-\)polarized components of the electric field, \(A \) is a constant related to the intensity of the beam, and

\[
I_m(r,z) = \int_0^\infty A_m(\theta)\cos \theta \sin \theta (1 + \cos \theta) f_m(kr \sin \theta) \exp(-ikz \cos \theta) \, d\theta,
\]

(5a)

\[
I_{m+1}(r,z) = \int_0^\infty A_m(\theta)\cos \theta \sin \theta (1 + \cos \theta) f_{m+1}(kr \sin \theta) \exp(-ikz \cos \theta) \, d\theta,
\]

(5b)

\[
I_{m+2}(r,z) = \int_0^\infty A_m(\theta)\cos \theta \sin \theta (1 + \cos \theta) f_{m+2}(kr \sin \theta) \exp(-ikz \cos \theta) \, d\theta,
\]

(5c)

here \(k=2\pi/\lambda \) is the wave vector, and \(\alpha = \arcsin(NA/n) \) is the maximum NA angle. Assuming the optical system is in free space (i.e., the refractive index is \(n=1\)), the maximal angle \(\alpha \) can be expressed as \(\alpha = \arcsin(NA/n) = \arcsin(NA)\). The coordinate system is shown in Fig. 1.

On substituting the expression \(A_m(\theta) \) of Eq. (2) into Eqs. (4) and (5), we can get the electric field distribution of inner ring in the double-vortex beams at the focal plane. In a similar way, we can also get the electric field distribution of \(y-\)polarized outer ring in the double-vortex beams. Finally, the total intensity distribution in the focal region can be obtained as follows:

\[
I(r,\phi,z) = E(r,\phi,z)E^*(r,\phi,z).
\]

(6)

From Eq. (6), we can numerically calculate the intensity distribution near the focus. In the following section, some numerical calculation results are given to show the focusing properties of the double-vortex beams through a high NA objective.

3. Results and discussions

Firstly, we focus on the intensity and phase properties of the double-vortex beams with different topological charges \(m_1\) and \(m_2\) in the focal plane. The fixed parameters for the calculations are \(\lambda=632.8 \text{ nm}, f=1 \text{ cm}, w_0=1 \text{ cm}, E_1=E_2=1\) and \(m_1=1\). Fig. 2 shows the total intensity distributions and its \(x-, y-\) and \(z-\)polarized components as the double-vortex beams with different topological charges of the outer ring are tightly focused. It is found that the intensity distribution of the \(x-\) and \(y-\)polarized components keep a hollow shape and the central intensity remains zero, but the core intensity of the \(z-\)polarized component is non-zero, which leads to the tiny dark core with non-zero central intensity in the total intensity distribution \[19\]. It is shown that the intensity has three field components in the focal region, indicating that the polarized double-vortex beams are depolarized when they are focused by a high NA objective. It is also found that the total intensity distributions are of double-ring structure and the gap between the inner and outer ring increase with increment of the topological charges of the outer ring. As \(m_2>5\), the double-ring begins to separate and becomes two bright rings. Comparing Fig. 2(a2) with (a3), we can see that the gap between the two rings in Fig. 2(a3) \(m_2=7\) is larger than that in Fig. 2(a2) \(m_2=5\).

The total intensity distribution and its \(x-, y-\) and \(z-\)polarized components in the focal plane are shown clearly in Fig. 2(e)–(g). It can be seen that the peak intensity of the outer ring increases and the space between the outer peaks becomes wider with the increment of the topological charges of the outer ring. As \(m_2=7\) (i.e., Fig. 2(g)), the total intensity distributions appear two double-peaks, and the inner and outer peak on the same side are separated about one \(\lambda\). It corresponds to the double-ring structure in Fig. 2(a3).

The phase contours of \(E_{x}(r,\phi,z), E_{y}(r,\phi,z)\) and \(E_{z}(r,\phi,z)\) with different topological charges of the outer ring in the focal plane are shown in Fig. 3. It is found that there is one phase singularity in the center of the phase contours of \(E_{x}(r,\phi,z)\). It is consistent with the topological charges of the inner ring of the double-vortex beams. And the phase contours of \(E_{y}(r,\phi,z)\) also reflect the numbers of topological charges of the outer ring (i.e., \(m_2\)). They all correspond to the intensity distribution in Fig. 2. It is concluded that each ring of the double-vortex beams is just the vortex beam with different topological charges.

Fig. 4 shows the contour distributions of the total intensity and its \(x-, y-\) and \(z-\)polarized components in the propagation plane for the double-vortex beams with \(m_1=1\) and \(m_2=7\). It can be seen that the intensity is at its maximum in the focal plane (i.e., \(z=0\)), and the intensity is decreased with the increase of distance apart from the focal plane. It is also shown that the central intensity of the \(x-\) and \(y-\)polarized components is zero, but there is a bright spot in the core of the \(z-\)polarized component. This result corresponds to Fig. 2(g).

Then we investigate the influence of NA on the focusing properties. The influence of varying NA on the total intensity in the focal plane for the double-vortex beams with \(m_1=1\), \(m_2=7\) is shown in Fig. 5. It is found that with the increase of NA, the characteristics of the intensity patterns are hardly changed, but more light field will be focused into the focal region, hence the intensity of the outer ring is gradually enhanced. Moreover, the focal spot size is decreased due to the tighter focused field with higher NA. As NA is chosen to be 0.95, the shape of the double-ring is clear and distinguishable.

4. Conclusions

In conclusion, we have studied the focusing properties of the double-vortex beams through a high NA objective based on
Fig. 2. Intensity distributions with different topological charges of the outer ring in the focal plane: (a1)–(d1) $m_2 = 3$; (a2)–(d2) $m_2 = 5$; (a3)–(d3) $m_2 = 7$. (a1)–(a3) the total intensity; (b1)–(b3) $I_x$ (the x-polarized component); (c1)–(c3) $I_y$ (the y-polarized component); (d1)–(d3) $I_z$ (the z-polarized component). The other parameters are chosen as $\lambda = 632.8$ nm, $f = 1$ cm, $w_0 = 1$ cm, NA = 0.95, $E_1 = E_2 = 1$ and $m_1 = 1$. 
the vectorial Debye theory. We focus on the intensity and phase properties in the focal plane and the influence of relative parameters on them. It is found that the total intensity distributions are of double-ring structure, and its shape can be changed by adjusting the topological charges or the NA of the focusing objective. Moreover, each ring of the double-vortex beams carries different orbital angular momentum, and is propagating independently. It is shown that the tight focusing of the double-vortex beams may find
applications in micro-particle trapping, manipulation, material processing, etc.

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