Cluster synchronization in fractional-order complex dynamical networks

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A B S T R A C T

Cluster synchronization of complex dynamical networks with fractional-order dynamical nodes is discussed in the Letter. By using the stability theory of fractional-order differential system and linear pinning control, a sufficient condition for the stability of the synchronization behavior in complex networks with fractional order dynamics is derived. Only the nodes in one community which have direct connections to the nodes in other communities are needed to be controlled, resulting in reduced control cost. A numerical example is presented to demonstrate the validity and feasibility of the obtained result. Numerical simulations illustrate that cluster synchronization performance for fractional-order complex dynamical networks is influenced by inner-coupling matrix, control gain, coupling strength and topological structures of the networks.

1. Introduction

In the past few decades, many efforts have been devoted to complex networks due to the wide and potential applications in various fields. Examples includes World Wide Web, communication networks, social organizations, food webs, power grid networks, genetic regulatory networks [1–5], and so on. Specifically, synchronization, as an important and interesting collective behavior of complex networks in our real life, has drawn increasing attention from different fields such as information science, biological system, image processing, secure communication, etc. Up to now, there are many widely-studied synchronization patterns, which define the correlated in-time behaviors among the nodes in a dynamical network, such as complete synchronization [6], phase synchronization [7,8], mixed synchronization [9,10], generalized synchronization [11,12], lag synchronization [15], cluster synchronization [16–19].

In particular, in many technological, social and biological networks, which can be divided naturally into communities, nodes in the same community often have the same type of function. Cluster synchronization is an exact phrase to describe this important and significant phenomenon. By general definition, the cluster synchronization is observed when the dynamical nodes synchronize with each other in each group formed by certain partition rules, but no synchronization appears between any two different groups. In recent years, some important and interesting results have been obtained. For example, Ma et al. discussed cluster synchronization of a connected chaotic networks and a star-like complex network in [18] and [20], respectively. Wang, et al. considered cluster synchronization in community networks with or without time delay [19]. Wu and Lu, and Lu and Qin investigated cluster synchronization in the adaptive complex dynamical networks in [13] and [21], respectively. Lu et al. and Wu et al. studied cluster synchronization in networks of coupled nonidentical dynamical systems in [22] and [23], respectively. Liu, Chen and Lu investigated the cluster synchronization problem for linearly coupled networks [24].

It is well known that the fractional calculus is a classical mathematical notion, and is a generalization of ordinary differentiation and integration to arbitrary (non-integer) order. However, the fractional calculus did not attract much attention for a long time due to lack of application background. Nowadays, studying fractional-order calculus has become an active research field. Researchers point out that many systems in interdisciplinary fields could be described by fractional differential equations, such as viscoelastic systems, dielectric polarization, electrode–electrolyte polarization and electromagnetic waves [25–28]. Compared with the classical integer-order models, fractional-order derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. Therefore, it may be more accurate to model by fractional-order derivatives than by integer-order ones. It is demonstrated that many fractional-order differential systems behave chaotically or hyperchaotically, such as...
a sufficient condition ensuring local cluster synchronization of the model is given. Finally, a numerical example is given to demonstrate the effectiveness of the derived criteria and corresponding numerical simulations show that the cluster synchronization performance is affected by inner-coupling matrix, feedback gains, coupling strength, and topological structures of the networks.

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