A new procedure for multi-mode sequential flocking with application to multiple non-holonomic mobile robot motion control: mode description and integration principle

Lei Cheng*

Engineering Research Center of Metallurgical Automation and Measurement Technology, Ministry of Education, Wuhan University of Science and Technology, Wuhan, 430081, China and Faculty of Computing, Engineering and Mathematical Sciences, University of the West of England, Bristol, BS161QY, UK E-mail: chenglei@wust.edu.cn *Corresponding author

Wen-Xia Xu and Huai-Yu Wu

Engineering Research Center of Metallurgical Automation and Measurement Technology, Ministry of Education, Wuhan University of Science and Technology, Wuhan, 430081, China E-mail: xwxsa1@163.com E-mail: whyjwc@163.com

Quan-Min Zhu

Faculty of Computing, Engineering and Mathematical Sciences, University of the West of England, Bristol, BS161QY, UK E-mail: quan.zhu@uwe.ac.uk

Yong-Ji Wang

Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, 430074, China E-mail: wangyjch@hust.edu.cn

Hassan Nouri

Faculty of Computing, Engineering and Mathematical Sciences, University of the West of England, Bristol, BS161QY, UK E-mail: Hassan.Nouri@uwe.ac.uk

Abstract: This is the first of two-part paper that investigates the multi-mode sequential flocking with application to multiple non-holonomic mobile robot motion control. To provide a new procedure to avoid collisions and obstacles in the process of multi-robot’s sequential flocking, this paper presents the design of a multi-model flocking control for multiple mobile robots in terms of behaviour-based robotics. Some nature-imitating behaviour modes are integrated into the new sequential flocking strategy, including single-robot potential-based behaviour, single-robot wall-following behaviour, multi-robot rigid-body bouncing behaviour, multi-robot path-tracking behaviour and their fusion state. In this way, the efficient collision and obstacle avoidance in flocking motion can be achieved.

Keywords: multi-robot; flocking control; multi-mode; sequential; behaviour-based mechanism.
1 Introduction

Multi-agent formation systems are a complex self-organising decentralised dynamic group which arises from the applications (Yang et al., 2009), including formation flight, advanced transportation systems, distributed sensors networks, search-and-rescue operations, and military surveillance. Flocking is a widespread natural phenomenon, flocking control is a new type of distributed motion control by simulation of the natural biological polymerisation. In recent years, the flocking control theory has caused great interest to the overseas research of intelligent systems.

A group consists of an indefinite number of individuals. In the group, the relative motion among individuals, and the whole group’s macroscopic motion in the external environment, both derived from the attraction and repulsion forces which exist in the ‘potential field’. ‘Potential field’ can be a real physical attraction/repulsion force, and it also can be a mathematical virtual force field- artificial potential function. Due to the different choices of ‘potential field’, the flocking behaviours are diverse.

Over the years, many algorithms to realise these rules have also been proposed, Tanner et al. (2003a, 2003b; Tanner, 2004, 2007; Saber et al., 2006) constructed local laws that allow a group of mobile agents moving with a common speed and achieve desired inter-agent distance while avoiding collisions with each other. Yu et al. (2005, 2006) investigated the flocking behaviours of multi-agent formations with a leader agent in two-dimensional plane. Through constructing a local control laws, consensus problem in multi-agent flocking system was researched by Olfati-Saber and Murray (2004) and Ren and Beard (2005). Li et al. (2009) present the control algorithm of obstacle...

The key feature of our proposed framework is the multi-mode flocking strategy, which integrates some natural-imitating behaviour modes with potential-based flocking mode as a whole. Such control scheme can be simply described as follows: when the robot group in common flocking suffer ‘potential trap’ and cannot move normally in the unexpected obstacle environment, it will autonomously switch to the suitable behaviour-based mode for continuously running. By this way, the unity of rapid convergent motion and efficient obstacle avoidance motion can be both achieved. In addition, to mimic the flying birds in nature, the leader(s) of flock is introduced. There are one or more leaders acting as pilot(s) in a group, and the other members follow the leader(s) orderly. This pilot mechanism in potential-based flocking was known as ‘sequential flocking’ (Cheng et al., 2008), which is the premise of our scheme.

Specially, there is another issue should be noticed. Multi-mobile robot formation is only an application of multi-agent coordination. Primary research activities (Saber, 2006; Saber and Murray, 2003; Tanner, 2004; Gazi and Passino, 2004), were focused on flocking control of agents. There is a difference between controlling agents and non-holonomic mobile robots: agents can sharply move to any headings smoothly due to the non-holonomic wheel constraints (Liang and Lee, 2006; Hanada et al., 2007). The universal dynamics transformation rules from agent to holonomic mobile robots: agents can sharply move to any headings smoothly due to the non-holonomic wheel constraints (Liang and Lee, 2006; Hanada et al., 2007). The universal dynamics transformation rules from agent to holonomic mobile robot in flocking were also part of our continuous investigation.

2 Backgrounds of multi-robot sequential flocking based on potential

2.1 Agent-based sequential flocking algorithm

The motion of group should be divided into two parts: leader’s motion and followers’ motion. Generally, the motion of the leader can be free, while the followers should strictly obey the following flocking theories. Consider \( N \) follower agents, moving on the plane with dynamics described by:

\[
\begin{align*}
\dot{\mathbf{r}}_i &= \mathbf{v}_i \\
\dot{\mathbf{v}}_i &= \mathbf{u}_i & i = 1, 2, \cdots, N.
\end{align*}
\]

where \( \mathbf{r}_i = (x_i, y_i)^T \) is the position vector of agent \( i \), \( \mathbf{v}_i = (\dot{x}_i, \dot{y}_i)^T \) is its velocity vector and \( \mathbf{u}_i = (u_{x_i}, u_{y_i})^T \) is control (acceleration) input.

The leader is self-driven at a velocity \( \mathbf{v}_L(t) \), the relative coordinate corresponding to the Leader \( (r_{Lx}, v_{Lx}, u_L) \). The position vector of the \( i \)-th agent relative to the leader is denoted \( \hat{r}_i = r_i - r_L \), and the velocity of the \( i \)-th agent relative to the leader is \( \hat{v}_i = v_i - v_L \). The motion equations becomes:

\[
\frac{d}{dt} \left( \begin{array}{c}
\hat{r}_i \\
\hat{v}_i
\end{array} \right) = \left( \begin{array}{c}
\hat{v}_i \\
\mathbf{u}_i - \dot{\mathbf{v}}_L
\end{array} \right) & i = 1, 2, \cdots, N
\]

Relative position vector between agent \( i \) and \( j \) is denoted \( \mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j \). Obviously, \( \| \mathbf{r}_{ij} \| = \| \mathbf{r}_{ji} \| \)

Considering:

\[
V_i = \sum_{j \in N_i} V_{ij} \left( \| \mathbf{r}_{ij} \| \right) = \sum_{j \in N_i} \chi_{N_i} (i) V_{ij} \left( \| \mathbf{r}_{ij} \| \right) + \sum_{j \in N_j \setminus N_i} V_{ij} \left( \| \mathbf{r}_{ij} \| \right)
\]

where \( \chi_{N_i} (i) = \begin{cases} 
1 & i \in N_L, \\
0 & i \notin N_L 
\end{cases} \), and \( N_i \) is neighbouring set of leader agent, representing the set of agents with which leader agent is allowed to communicate or sense.

Artificial potential function \( V_{ij} \) is a differentiable, non-negative, radially unbounded function of the distance \( \| \mathbf{r}_{ij} \| \) between agents \( i \) and \( j \), \( r_{ij} = r_i - r_j \). (Tanner et al., 2003a)

\( V_{ij} \) satisfies:

\begin{enumerate}
\item \( V_{ij} \left( \| \mathbf{r}_{ij} \| \right) \to 0 \) as \( \| \mathbf{r}_{ij} \| \to 0 \),
\item \( V_{ij} \) attains its unique minimum when agents \( i \) and \( j \) are located at a specified distance.
\end{enumerate}

The control law \( \mathbf{u}_i \) can then be defined as:

\[
\mathbf{u}_i = \frac{-\sum_{j \in N_i} \nabla V_{ij} \left( \| \mathbf{r}_{ij} \| \right) - \hat{v}_L}{\alpha_i}
\]

where \( \alpha_i \) is a vector that aligns the velocity of all the agents and makes them move with a common speed and direction and the variable \( \alpha_i \) is a vector in the direction of the negated gradient of an artificial potential function.

\textbf{Theorem 1:}

Consider a system of \( N \) mobile agents with dynamics (1), each agent is steered by control law (4) and assume that the neighbouring graph is connected (note: means no agent is isolated to the group). Then all agent velocity vectors become asymptotically the same, relative distance between agents maintain constant, collisions between interconnected agents are avoided.

\textbf{Remark 1:}

\( \mathbf{u}_i \) can be rewritten as:

\[
\mathbf{u}_i = -A \sum_{j \in N_i} \nabla V_{ij} \left( \| \mathbf{r}_{ij} \| \right) - \hat{v}_L + \hat{v}_L
\]
where, \( A > 0, A \) is gradient adjustment coefficient, which is used for balance between \( a_i \) and \( a_c \). \( u_i \) can also make the flock stable.

**Remark 2:**
\( V_y \) can be selected as:
\[
V_y = \frac{L^2}{F_y} + \log |F_y|^2
\]
(6)
where, \( L \) is configuration coefficient, which can determine the structure of the flock.

### 2.2 Overall control law transformation from agent to mobile robot

The control law in (4) has been used for controlling agents in flocking. Because of mobile robots’ unidirectional wheel constraints, there should be a bridge between agent and non-holonomic mobile robot. The guiding principle of control transformation from agents to mobile robots is: transforming an agent’s acceleration input \( u_i \) to a mobile robot’s translational velocity input \( v_i \) and rotational velocity input \( \omega_i \).

From expressions of \( u_i \) in (4) and (5), \( u_i \) can be considered as the virtual force operated on the mobile robot. Figure 1 illustrates a mobile robot controlled by \( u_i \).

**Figure 1** Mobile robot controlled by virtual force \( u_i \)

In Figure 1, \( R \) is the radius of the mobile robot, \( p_{rp} \) is the reference point of the mobile robot, a point along the robot’s axis line, \( p_c \) is the rotation centre of the mobile robot. The displacement between \( p_c \) and \( p_{rp} \) is \( d \). The positive direction of \( d \) is along the front direction of the robot. The attitude of the robot is represented by \((x_{prc}, y_{prc}, \theta_i)\), \( \theta_i \) is the heading of the mobile robot, \( u_i \) is decomposed into \( u_{ix} \) and \( u_{iy} \), and \( u_{iy} \) is vertical to \( u_{ix} \). The direction of \( u_i \) is represented by \( \beta \). The positive direction of \( u_{ix} \) is also along the front direction of the robot, while the positive direction of \( u_{iy} \) is counterclockwise to \( p_{rc} \).

**Theorem 2:**
From principles of mechanics, equation (7) represents the control transformation from agent-based system \((u_i)\) to mobile robots in flocking motion \((v_i(t + \Delta t), \omega_i(t + \Delta t))\).

\[
v_i(t + \Delta t) = \frac{u_i \cdot \cos(\beta_i - \theta_i)}{m_i} \cdot \Delta t + v_i(t) \quad (7a)
\]
\[
\omega_i(t + \Delta t) = \frac{u_i \cdot \sin(\beta_i - \theta_i) \cdot d}{IR_{zz_i}} \cdot \Delta t + \omega_i(t) \quad (7b)
\]
In (7), \( v_i(t) \) and \( \omega_i(t) \) are the current translational velocity and rotational velocity, respectively, \( v_i(t + \Delta t) \) and \( \omega_i(t + \Delta t) \) represents the translational velocity input \( v_i \) and rotational velocity input \( \omega_i \) in the next cycle, respectively. \( m_i \) is defined as the virtual mass of the robot, and \( IR_{zz_i} \) is defined as the virtual moment of inertia of the robot.

### 3 Multi-mode sequential flocking with collision and obstacle avoidance

Substantially, the algorithms of ‘multi-robot sequential flocking’ in Section 2 were applied to obstacle-free zone, which advantage lies on autonomous group aggregation and stable formation. To be followed up, it is necessary to explore the applicable ‘sequential collision and obstacle avoidance flocking’ strategy which is applicable to any environment. Yet, the multi-robot obstacle avoidance motion purely relied on potential principle will get into the ‘potential trap’, namely ‘dead-lock zone’, due to the local minimum problem. Relatively, solving the real-time control problem in complex obstacle environment is just the advantage of behaviour-based robotics. In this part, the main point of our research, multi-mode flocking strategy will be presented. Such control scheme can be simply described as follows: when the robot group in common flocking suffers ‘potential trap’ and cannot move normally in the unexpected obstacle environment, it will autonomously switch to the suitable behaviour-based mode for continuously running. By this way, the unity of rapid convergent motion and efficient obstacle avoidance motion can be both achieved. Various behaviour modes accompanied with multi-robot sequential flocking include single-robot potential-based behaviour, single-robot wall-following behaviour, multi-robot rigid-body bouncing behaviour, multi-robot path-tracking behaviour and their mutual integration.

#### 3.1 Single-robot potential-based behaviour mode

In multi-mode sequential flocking, the function of single-robot potential-based behaviour mode is of the general exploration for the leader in obstacle environment. Artificial potential field method is a common method used in robot’s motion control, and the virtual force makes the robot go through the obstacles and move towards the goal (Chen et al., 2003).

**3.1.1 Repulsive potential function**

This potential field is similar to electric potential field, potential energy and distance are inversely proportional, and the repulsive potential function can be chosen as:
A new procedure for multi-mode sequential flocking

\[ U_r = \begin{cases} \frac{k_1}{d} & d \leq d_{\text{max}} (d \neq 0) \\ 0 & d > d_{\text{max}} \end{cases} \] (8)

where \( d \) is the distance between robot’s measuring point and the obstacles, \( d_{\text{max}} \) is the maximum extent that the potential field effects, \( k_1 \) is the repulsion factor. The robot’s subjected repulsion is:

\[ F_r = -\nabla U_r = \begin{cases} \frac{k_1}{d^2} & d \leq d_{\text{max}} (d \neq 0) \\ 0 & d > d_{\text{max}} \end{cases} \] (9)

The angle of repulsion is as:

\[ \theta_r = \pi + \arctan \left( \frac{y_{\text{obst}} - y_{\text{robot}}}{x_{\text{obst}} - x_{\text{robot}}} \right) \] (10)

where \((x_{\text{obst}}, y_{\text{obst}})\) is the obstacle point location coordinate, \((x_{\text{robot}}, y_{\text{robot}})\) is the robot location coordinate.

### 3.1.2 Attractive potential function

The target point on the robot's potential function is also based on distance. Attractive potential function can be chosen as:

\[ U_a = k_2 d \] (11)

where \( k_2 \) is the gravitational coefficient, \( d \) is the distance between the robot and the target point. Robot’s subjected gravitation is as:

\[ F_a = \nabla U_a = k_2 \] (12)

The angle of gravitation is as:

\[ \theta_a = \arctan \left( \frac{y_{\text{goal}} - y_{\text{robot}}}{x_{\text{goal}} - x_{\text{robot}}} \right) \] (13)

\((x_{\text{goal}}, y_{\text{goal}})\) is the location coordinate of the target point.

**Figure 2** The robot’s subjected force situation in obstacle environment (see online version for colours)

The robot’s subjected force situation in the obstacle environment is shown in Figure 2. Each reflected laser point represents an obstacle point, and scanning laser beams convert the detected obstacles’ information into repulsion \( F_r \). The distance information between target point and robot is converted into attraction \( F_a \). The composite vector \( F \) controls robot’s translational acceleration and rotational acceleration, to achieve the purpose of motion programming.

Because \( F \) is a ‘virtual force’ generated by artificial potential field, therefore, \( F \) is equivalent to \( u_i \) in Section 2. By using equation (7), the translational and rotational speed for mobile robot can be obtained.

### 3.2 Single-robot wall-following behaviour mode

Single-robot wall-following behaviour mode is suitable for leader as well. So-called wall-following behaviour can be explained as follows: the robot moves along the outline of obstacle, while maintaining specific distance from it. The combination of single robot’s wall-following and potential-based behaviour mode can help the leader in flock to overcome the random obstacles.

#### 3.2.1 Virtual accompanying robot

Referring to the idea ‘\( \alpha \) agent’ (Saber and Murray, 2003), we propose the concept of ‘virtual accompanying robots’ to interpret the mobile robot’s wall-following behaviour, as shown in Figure 3.

**Figure 3** Indication of virtual accompanying robot

As shown in Figure 3, virtual accompanying robot is the ‘projection’ of entity robot on the obstacle. The actual location of the virtual accompanying robot can be as follows: the coordinate of the shortest returning laser points when the entity robot detects obstacles (The laser point is similar to the entity robot’s orthogonal projection on the obstacles’ surface); the virtual accompanying robot’s heading is the obstacles’ surface (wall) angle. In the wall-following behaviour, the ‘repulsion’ effect of the virtual accompanying robot makes its ‘mother’ robot keep a certain distance from itself (collision avoidance) and the same angle (walk along the wall). Therefore, the obstacle avoidance motion of the entity robot combining with wall-following behaviour can be seen as: the process of a ‘repulsion’ of virtual accompanying robot works with gravitation of the target point.

The basic principle of integration with potential-based behaviour mode and wall-following behaviour mode can be simply demonstrated in Figure 4. Corresponding algorithm is briefly listed as following, much more details was shown in our contribution (Zhao and Fang, 2006).
Algorithm description

- **Step 1:** Judgement of virtual accompanying robot’s generation
  If the robot is trapped into ‘local minimum’ in the process of obstacle avoidance with potential-based behaviour mode, it will stop or produce a certain oscillation, namely deadlock. After deadlock having formed, virtual accompanying robot is generated. Virtual accompanying robot’s coordinate is the shortest reflected laser point coordinate when the entity robot detects the obstacles (as seen in Figure 3), and its heading is the angle of ‘wall’. At the moment, the distance between entity robot and it’s virtual accompanying robot is the initial obstacle distance $s_{0}$.

- **Step 2:** Motion with wall-following behaviour mode
  When the robot gets into the state of wall-following behaviour, it will move along the ‘wall’ with a constant speed $v_{wf}$. In the movement process, the robot continuously detects the distance $d_s$ between itself and its virtual accompanying robot. If $d_s < s_{0}$, the robot will rotate with a ‘repulsive’ angle $\Delta \theta_s$, to deviate from dangerous area. After having left dangerous area, the robot will go on moving along the ‘wall’ according to above principles.

- **Step 3:** Judgement for virtual accompanying robot’s disappearance
  In the process of robot wall-following motion, it still continuously makes the judgment whether its virtual accompanying robot disappears or not. When the conditions are satisfied, the robot will enter into the potential-based behaviour motion mode.

The conditions for the virtual accompanying robot’s disappearance are:

1. the entity robot can not detect any obstacles within any part of the detectable areas
2. the angle between ‘repulsion’ direction of the virtual accompanying robot and ‘gravitation’ direction of the target point is an acute angle.

At this moment, the role of the virtual accompanying robot’s ‘repulsion’ is the same with that of the target point’s ‘gravitation’, both of them make the entity robot move towards the target point. Meanwhile, although the entity robot can still detect obstacles, it will depart from its surface and move towards the target point.

- **Step 4:** Motion with potential-based behaviour mode
  After entering this state, robot will move towards the target point again. If the robot encounters obstacles again in the process, it will repeat the cycle Step1-Step3.

Further simulated outcome from the study is shown in Figure 5 and Figure 6 to demonstrate the process of wall-following behaviour with single mobile robot in obstacle environment.

In each graph, the mobile robots are indicated with directed circles, which radius is $R = 10mm \sqrt{a^2 + b^2}$. In Figure 5, the hollow circle represents that the robot was on the wall-following behaviour mode. Oppositely, the solid one represents that the robot was on the potential-based behaviour mode, for example, in Figure 6, the robot was attracted by goal. In the wall-following behaviour mode, $d_s = 30 $mm, $v_{wf} = 100$ mm/s, and in the potential-based behaviour mode, $k_1 = 1,000$, $k_2 = 1,000$. 

**Figure 5** The robot in the single-robot wall-following behaviour mode (see online version for colours)

**Figure 6** The robot in the single-robot potential-based behaviour mode (see online version for colours)
3.3 Multi-robot rigid-body bouncing behaviour mode

In the multi-robot motion process, the group members may suffer mutual collisions in some circumstance. If the relative distance and velocity between the robots can be known, it is possible to convert the collision problem between the robots into the rigid bodies bouncing issue. We will consider the impact of the relative velocity between the robots, dealing with the collision avoidance between robots by using the ‘virtual rigid-body contact force’ concept. Comparing to the pure potential field collision avoidance or behaviour-based collision avoidance (Balch and Archim, 1998), the algorithm brings in speed factors. In the case of the same relative distance, the robot with a larger relative velocity will obtain greater repulsion.

3.3.1 Rigid-body contact model

Song described the contact models between the rigid bodies in the literature (Song et al., 2001). If not taking rigid body’s contacting friction into account, there is the following representation model, as shown in Figure 7.

![Figure 7 Rigid-body contact model](image)

Rigid body can be divided into the core layer and deformed layer, rigid-body deformation layer consists of the parallel springs and dampers. is the contact point deformation along the normal direction before and after deformation. Set as the contact force of contact point along the normal direction, the following formula holds:

\[ \delta_N = f(\delta_N') + g(\delta_N', \delta_N) \]  

(14)

where \( f() \) and \( g() \) respectively represent the elasticity function and the damping function, the simplest representation of the both is Kelvin-Voigt model:

\[ f = K \cdot \delta_N, \quad g = C \cdot \delta_N' \]  

(15)

where \( K \) is the rigidity factor, \( C \) is the damping factor.

3.3.2 Algorithm description:

Chaimowicz put the above models in the contact-type motion coordination and collaboration of multi-robot (such as moving objects with collaboration) (Chaimowicz, 2002). We make it into the use of non-contact multi-robot collision avoidance movement.

Supposing maximum radius of robot is \( R \), safe collision avoidance distance is \( D \), the radius about the robot is \( R + \frac{D}{2} \). (\( x_i, y_i, \theta_i, v_i \)) is the location coordinate of robot \( i \), and (\( x_j, y_j, \theta_j, v_j \)) is the location coordinate of robot \( j \), if the \( i \)th and \( j \)th virtual rigid body occur ‘collision’, the deformation of \( i \)th virtual rigid body is \( \delta_N^i \), there:

\[ \delta_N^i = (2R + D) - \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \]  

(16)

where, \( \delta_N^i > 0 \), and the angle of contact point’s normal direction is: \( \beta_N = \arctan \left( \frac{y_j - y_i}{x_j - x_i} \right) \).

\( \delta_N^i \) is equivalent to the relative speed’s component in the contact point’s normal direction:

\[ \delta_N^i = v_j \cos(\theta_j - \beta_N) - v_i \cos(\theta_i - \beta_N) \]  

(17)

From equations (14)–(17), the force \( f' = K \cdot \delta_N^i + C \cdot \delta_N' \) of the robot in the virtual rigid body contact points can be obtained, which is the driving force for collision avoidance between robots and will be used in the next part ‘multi-robot path-tracking behaviour mode’.

3.4 Multi-robot Path-tracking behaviour mode

Through observing and analysing animal passing the obstacles by lining-up strategy (for example, the herd pass through the crawlway in sequence), it is reasonable to put forward multi-robot path-tracking behaviour mode in obstacle environments, as a unique and valid obstacle avoidance measure. The fundamental idea is: all of the followers should form an ordered queue at first and pass through obstacles one by one, guided by the leader. In the process of motion, the position of former member will be ‘movement attractor’ of succeeding member. The whole team will obey such rule, and go forward in the manner of ‘creeping’. After all members pass through the obstacles, the multi-robot path-tracking behaviour mode guided by leader can be automatically switched to original sequential flocking mode without obstacles.

Such idea converts relatively complex multi-robot obstacles avoidance problem into ordinary single-robot’s issue. If the local minimum problem can be properly resolved in the single robot’s motion, it is possible to expand the solution to that of robot groups. As described above, one of the feasible solutions to single-robot obstacle avoidance in any situation is the integration of single-robot wall-following behaviour mode with its potential-based motion mode described in above sections. To sum up, such multi-mode strategy makes multi-robot flocking with obstacles simple and effective.

The corresponding algorithm is described as following:

- **Step1:**

  The group leader robot detect whether obstacle exist in its front environment or not via its sensor system (for example: laser range finder). If not, the group maintains initial sequential flocking state. Otherwise, the leader begins switching to wall-following behaviour mode,
and the followers are ready for switching to multi-robot path-tracking behaviour mode. Then, goes into Step 2.

- **Step 2:**
  Sort out the followers according to their distances to the leader. For instance, if leader is 0#, the nearest follower is 1#, and so on. Then, goes into Step 3.

- **Step 3:**
  Construct path-tracking chain of the robot group. This is the most important step. Here, a typical three-robot example is selected to illustrate the whole algorithm. The dynamic process is also demonstrated in Figure 8.

**Figure 8** Establishment process of path-tracking chain in Step 3

In Figure 8, 0# robot is leader, 1# and 2# robot are both followers, and Track[0] and Track[1] are both position pointers of robot. The function of position pointer will be discussed in following part.

- **Step 3.a:**
  After having switched into the multi-robot path-tracking behaviour mode, the position pointers of the ith (i#) robot, Track[0] and Track[1], both point to the robot’s current position.

- **Step 3.b (the leader’s strategy):**
  The leader (0# robot, i = 0) begins moving. When $S_0$, the distance between the current position of leader and the position which leader’s Track[0] points at, is larger than set value $P$, leader’s Track[0] redirects to its current moving position. Meanwhile, the leader’s Track[1] redirects to the position which leader’s Track[0] originally points at. Furthermore, the position which the leader’s Track[1] points at becomes the ‘movement attractor’ of subsequent follower (1# robot, also is follower), described by $(x_{ma}, y_{ma})$.

- **Step 3.c (the follower’s strategy):**
  When $i = 1$, the first follower, 1# robot, moves to the position which the 0# robot’s Track[1] points at, in the manner of ‘attracting movement’. When $S_1$, the distance between the current position of 1# robot and the position which 1# robot’s Track[0] points at, is larger than set value $P$, 1# robot’s Track[0] redirects to its current moving position. Meanwhile, the 1# robot’s Track[1] redirects to the position which 1# robot’s Track[0] originally points at. Furthermore, the position which the 1# robot’s Track[1] points at becomes the ‘movement attractor’ of subsequent follower (2# robot, also is follower), described by $(x_{ma}, y_{ma})$.

And then, when $i > 2$, the algorithm is same as Step3.c. The Track[0] and Track[1] of all group members constitute the path-tracking chain in multi-robot queue motion. The $(i + 1)$th robot moves to the $i$th robot in the manner of ‘attracting movement’ continuously. The ‘attracting movement’ can be realised by potential principle or constant speed motion. Overall, the robot group’s moving shows a vivid worm-like ‘creeping’ behaviour.

During the switching process from sequential flocking mode to path-tracking mode, it is possible to see that various robots collide with each other owing to their path conflict. The rigid-body bouncing behaviour mode between robots is employed here. The speed of mobile robot in this behaviour mode is a constant value represented by $v_{rb}$. The corresponding algorithm is described as below:

If $f$ (produced in rigid-body bouncing behaviour mode) of the $i$th robot [described by $(x_i, y_i, \theta_i)$] is not equal to 0, $v_{rb} = 0$. In the whole group dynamic motion, when $\arctan \left( \frac{y_{ma} - y_i}{x_{ma} - x_i} \right) - \arctan \left( \frac{y_{ma} - y_{ma}}{x_{ma} - x_{ma}} \right) \leq \pi / 2$, $v_{rb}$ turns into const, and $\theta_i = \arctan \left( \frac{y_{ma} - y_i}{x_{ma} - x_i} \right)$. The $i$th robot moves to its ‘movement attractor’, while collision eliminates.

- **Step 4:**
  After all members of group have passed the obstacle environment, multi-robot path-tracking behaviour mode is automatically switched to sequential flocking mode without obstacles, as illustrated in Figure 9.

**Figure 9** State switching graph of sequential flocking mode and path-tracking behaviour mode

4 Conclusions

This study has illustrated how a group of non-holonomic mobile robots can cooperate with ad hoc sequential flocking behaviour in the irregular obstacles environment. The integrated multi-robot rigid-body bouncing collision and path-tracking obstacle avoidance strategy is presented to construct a class of real-time and environment-adaptive multi-robot flocking systems. By adopting multi-mode
switching control mechanism, the stable formation and flexible/efficient obstacles avoidance capability are both accommodated in such a multi-robot flocking system.

Acknowledgements
This work was supported in part by the National Science Foundation in China under Grant No. 60705035 and 61075087, this is greatly acknowledged.

References


