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Available online: 02 Nov 2011

To cite this article: Yang Lei, Shurong Li, Xiaodong Zhang, Qiang Zhang & Lanlei Guo (2011): Optimal control of polymer flooding based on mixed-integer iterative dynamic programming, International Journal of Control, 84:11, 1903-1914

To link to this article: http://dx.doi.org/10.1080/00207179.2011.629321

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Optimal control of polymer flooding based on mixed-integer iterative dynamic programming

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(Received 1 July 2011; final version received 1 October 2011)

Polymer flooding is one of the most important technologies for enhanced oil recovery. In this article, a mixed-integer optimal control model of distributed parameter systems (DPS) for the injection strategies is established, which involves the performance index as maximum of the profit, the governing equations as the fluid flow equations of polymer flooding and some inequalities constraints, such as polymer concentration and injection amount limitation. The control variables are the volume size, the injection concentration of each slug and the terminal flooding time. For the constant injection rate, the slug size is determined by the integer time stage length, and thus the integer variables are introduced in the DPS. To cope with the optimal control problem (OCP) of this DPS, a mixed-integer iterative dynamic programming incorporating a special truncation procedure to handle integer restrictions on stage lengths is proposed. First, the OCP with variable time stage lengths is transformed into a fixed time stage problem by introducing a normalised time variable. Then, the optimisation procedure is carried out at each stage and preceded backwards in a systematic way. Finally, the numerical results of an example illustrate the effectiveness of the proposed method.

Keywords: optimal control; polymer flooding; iterative dynamic programming; mixed-integer

1. Introduction

It is of increasing necessity to produce oil fields more efficiently and economically because of the ever-increasing demand for petroleum worldwide. Since most of the significant oil fields are mature fields and the number of new discoveries per year is decreasing, the use of enhanced oil recovery (EOR) processes is becoming more and more imperative. At present, polymer flooding technology is the best method for chemically EOR (Qing et al. 2010). It could reduce the water–oil mobility ratio and improve sweep efficiency (Maitin 1992; de Melo et al. 2005; Yu, Jiang, and Zhao 2010; Jiang, Yu, and Yi 2011).

Due to the high cost of chemicals, it is essential to optimise polymer injection strategies to provide the greatest oil recovery at the lowest cost. The optimisation procedure involves maximising the objective function (cumulative oil production or profit) from a polymer flooding reservoir by adjusting the injection concentration and volume size of every slug, as well as the flooding terminal time. One way of solving this problem is direct optimisation with the reservoir simulator. Numerical models are used to evaluate the complex interactions of variables affecting development decisions, such as reservoir and fluid properties and economic factors. Even with these models, the current practice is still the conventional trial and error approach. In each trial, the injection polymer concentration or volume size for one slug is selected based on the intuition of the reservoir engineer. This one-variable-at-a-time approach may lead to suboptimal decisions because engineering and geologic variables affecting reservoir performance are often nonlinearly correlated, and the problem definitely compounds when multiple producers and injectors are involved in a field development case. The use of the optimal control method offers a way out.

The optimal control method has been researched in EOR techniques in the recent years. Ramirez, Fathi, and Cagnol (1984) first applied the theory of optimal control to determine the best possible injection strategies for EOR processes. Their study was motivated by the high operation costs associated with EOR projects. The objective of their study was to develop an optimisation method to minimise injection costs while maximising the amount of oil recovered. The performance of their algorithm was subsequently examined for surfactant injection as an EOR process in a one-dimensional core flooding problem (Fathi and Ramirez 1985). The control for the process was the surfactant concentration of the injected fluid. They observed a significant improvement in the ratio of the value of the oil recovered to the cost of the surfactant injected from 1.5 to about 3.4. Optimal control was

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ISSN 0020–7179 print/ISSN 1366–5820 online
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http://dx.doi.org/10.1080/00207179.2011.629321
http://www.tandfonline.com
also applied to steam flooding by Liu and Ramirez (1993). They developed an approach using optimal control theory to determine the operating strategies to maximise the economic attractiveness of steam flooding process. Their objective was to maximise a performance index which is defined as the difference between oil revenue and the cost of injected steam. Their optimisation method also obtained significant improvement under optimal operation. Ye, Qi, and Fang (1998) were involved in the study of optimal control of gas-cycling in condensate reservoirs. It was shown that both the oil recovery and the total profit of a condensate reservoir can be enhanced obviously through optimisation of gas production rate, gas injection rate and the mole fractions of each component in injection gas. Brouwer et al. (2002) and Sarma et al. (2005) used the optimal control theory as an optimisation algorithm for adjusting the valve setting in smart wells of water flooding. The water flooding scheme that maximised the profit was numerically obtained by combining reservoir simulation with control theory practices of implicit differentiation. They were able to achieve improved sweep efficiency and delayed water breakthrough by dynamic control of the valve setting.

For the previous work on optimal control of polymer flooding, Li and Zhang (2008), Zhang and Li (2008) and Zhang, Zhang, Lei, Li, and Zhou (2010) presented an adjoint method based on Pontryagin maximum principle for computation of the gradient of the objective function with respect to the controls. As a result of the large and complicated nature of reservoir models with nonlinear constraints, the software for gradient calculations for practical optimisation programs are very tedious and time consuming to create. Li, Lei, Zhang, and Zhang (2010) and Lei, Li, and Zhang (2010) used the genetic algorithms to determine the optimal injection strategies of polymer flooding, and the reservoir model equations were treated as a ‘black box’. The genetic algorithms are capable of finding the global optimum on theoretical sense, but as Sarma, Aziz, and Durlufski (2005) point out that they require tens or hundreds of thousand reservoir simulation runs of very large model and are not able to guarantee monotonic maximisation of the objective function. To avoid these difficulties, Guo, Li, Zhang, and Lei (2009) applied the iterative dynamic programming (IDP) algorithm to solve the optimal control problem (OCP) of a one-dimensional core polymer flooding. Through the usage of coarse grid points and region-reduction strategy, IDP not only promotes the efficiency of computation but also increases the numerical accuracy. However, the optimal control model used in their study is so simple that it is not adaptable for practical oilfield development.

Furthermore, all the methods above only optimised the injection polymer concentration, but did not consider the slug size and terminal time as optimisation variables.

In this article, an optimal control model of distributed parameter systems (DPS) for polymer flooding is established which maximises the profit by adjusting the injection concentration, the slug size and the flooding terminal time. The optimisation of slug size is transformed to determine the time stage length which is an integer variable in days. Then, the determination of polymer injection strategies turns to solve this mixed-integer OCP of the DPS. A mixed-integer IDP algorithm in combination with the full implicit finite-difference simulator is proposed for the OCP. Finally, the method is tested with a polymer flooding project involving a heterogeneous reservoir case. The example demonstrates that the profit gained by OCP is higher than that by trial and error solutions.

2. Mathematical formulation of optimal control

2.1 Performance index

Let \( \Omega \in \mathbb{R}^2 \) denote the domain of reservoir with boundary \( \partial \Omega \), \( n \) be the unit outward normal on \( \partial \Omega \) and \( (x, y) \in \Omega \) be the coordinate of a point in the reservoir. Given a free terminal time \( t_f \), we set \( \Psi = \Omega \times (0, t_f), \quad \Sigma = \partial \Omega \times (0, t_f) \); suppose that there exist \( N_w \) injection wells and \( N_o \) production wells in the oilfield. The injection and production wells are located at \( L_w = \{(x_{wi}, y_{wi}) | i = 1, 2, \ldots, N_w \} \) and \( L_o = \{(x_{oi}, y_{oi}) | j = 1, 2, \ldots, N_o \} \), respectively. This descriptive statement of the cost functional must be translated into a mathematical form to use quantitative optimisation techniques. The oil value can be formulated as

\[
\int_0^{t_f} \int_\Omega \xi_o (1 - f_w) q_{out} \, d\sigma \, dt, \tag{1}
\]

where \( \xi_o \) is the cost of oil per unit mass (10^4 $/m^3), \( f_w(x, y, t), (x, y) \in L_o \), the water cut of production well, \( 1 - f_w \) the oil cut of production well and \( q_{out} \) the flow velocity of production fluid (m/day).

The polymer cost is expressed mathematically as

\[
\int_0^{t_f} \int_\Omega \xi_p q_{in} c \, d\sigma \, dt, \tag{2}
\]

where \( \xi_p \) is the cost of oil per unit volume (10^4 $/kg), \( q_{in} \) the flow velocity of injection fluid (m/day) and \( c \) the polymer concentration of the injection fluid (g/L).
The performance index is defined as the following functional,
\[
\max J = \int_0^T \int_\Omega [\xi_0(1 - f_w)q_{out} - \xi_p q_{in} c_{pw}] \, d\sigma \, dt. \tag{3}
\]

2.2 Governing equations

The maximisation of the cost functional \( J \) given by Equation (3) is not totally free but is constrained by the system process dynamics. The governing equations of the polymer flooding process must therefore be developed to describe the flow of both the aqueous and oil phases through the porous media of a reservoir formation. The equations used in this article allows for the adsorption of polymer onto the solid matrix in addition to the convective and dispersive mechanisms of mass transfer. Let \( p(x, y, t), S_w(x, y, t) \) and \( c_p(x, y, t) \) denote the pressure, water saturation and polymer concentration of the oil reservoir, respectively, in a point \((x, y) \in \Omega \) and a time \( t \in [0, t_f] \), then \( p(x, y, t), S_w(x, y, t) \) and \( c_p(x, y, t) \) satisfy the following partial differential equations (PDEs):

- The flow equation for oil phase
  \[
  \frac{\partial}{\partial x} \left( \frac{K_{krw} h}{B_o \mu_o} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{K_{krw} h}{B_o \mu_o} \frac{\partial p}{\partial y} \right) - q_o = h \frac{\partial}{\partial t} \left( \frac{1 - S_w}{B_o} \right), \quad (x, y, t) \in \Psi. \tag{4}
  \]

- The flow equation for water phase
  \[
  \frac{\partial}{\partial x} \left( \frac{K_{krw} h}{B_w R_k \mu_p} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{K_{krw} h}{B_w R_k \mu_p} \frac{\partial p}{\partial y} \right) - q_w = h \frac{\partial}{\partial t} \left( \frac{S_w}{B_w} \right), \quad (x, y, t) \in \Psi. \tag{5}
  \]

- The flow equation for polymer component
  \[
  \frac{\partial}{\partial x} \left( D_h \frac{\phi_p S_w c_p}{B_w} \frac{\partial c_p}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_h \frac{\phi_p S_w c_p}{B_w} \frac{\partial c_p}{\partial y} \right) + \frac{\partial}{\partial y} \left( \frac{K_{krw} h c_p}{B_w R_k \mu_p} \frac{\partial c_p}{\partial y} \right) - q_w c_{pw} = h \frac{\partial}{\partial t} \left[ \frac{\phi_p S_w c_p}{B_w} + \rho_i (1 - \phi) C_{rp} \right], \quad (x, y, t) \in \Psi. \tag{6}
  \]

- The boundary conditions and initial conditions
  \[
  \left. \frac{\partial p}{\partial n} \right|_\Omega = 0, \quad \left. \frac{\partial S_w}{\partial n} \right|_\Omega = 0, \quad \left. \frac{\partial c_p}{\partial n} \right|_\Omega = 0, \quad (x, y, t) \in \Sigma, \tag{7}
  \]

Moreover, the polymer solution viscosity \( \mu_p \) (mPa s), the permeability reduction factor \( R_k \) and the amount adsorbed per unit mass of the rock \( C_{rp} \) (mg/g) which depend on the polymer concentration \( c_p \) are given by

\[
\mu_p = \mu_w [1 + (ap_1 c_p + ap_2 c_p^2 + ap_3 c_p^3)], \tag{15}
\]

\[
R_k = 1 + \left( \frac{R_{k_{max}} - 1}{1 + b_{k} \cdot c_p} \right), \tag{16}
\]

\[
C_{rp} = \frac{a c_p}{1 + b c_p}, \tag{17}
\]

\( K \) is the absolute permeability \((\mu m^2)\), \( h \) the thickness of the reservoir bed \((m)\), \( D \) the diffusion coefficient of polymer \((m^2/s)\), \( \rho_i \) \((kg/m^3)\) the rock density and \( \mu_o \) \((mPa s)\) the oil viscosity.

The oil volume factor \( B_o \), the water volume factor \( B_w \), the rock porosity \( \phi \) and the effective porosity to polymer \( \phi_p \) are expressed as functions of the reservoir pressure \( p \):

\[
B_o = B_{o_{ref}}[1 + C_o(p - p_r)], \tag{9}
\]

\[
B_w = B_{w_{ref}}[1 + C_w(p - p_r)], \tag{10}
\]

\[
\phi = \phi_{ref}[1 + C_R(p - p_r)], \tag{11}
\]

\[
\phi_p = f_o \phi, \tag{12}
\]

where \( p_r \) is the reference pressure \((MPa)\), \( \phi_{ref}, B_{o_{ref}} \) and \( B_{w_{ref}} \) denote the porosity, the oil and water volume factors under the condition of the reference pressure, respectively, \( f_o \) is the effective relative permeability at the irreducible water saturation \( S_{w_{irr}} \), \( k_{r_{o_{ref}}} \) is the oil relative permeability at the irreducible oil saturation \( S_{o_{irr}} \), \( n_w \) and \( n_o \) are the constant coefficients.
where $\mu_w$ is the viscosity of the aqueous phase with no polymer (mPa s) and $q_{p1}$, $q_{p2}$, $q_{p3}$, $R_{k_{\text{max}}}$, $b_{rk}$, $a$ (cm$^3$/g) and $b$ (g/L) are the constant parameters.

The source terms of Equation (4)–(6) are defined as

$$q_0 = \begin{cases} (1 - f_w)q_{\text{out}}, & (x, y) \in L_o, \\ 0, & (x, y) \notin L_o, \end{cases} \quad (18)$$

$$q_w = \begin{cases} f_w q_{\text{out}}, & (x, y) \in L_o, \\ -q_{\text{in}}, & (x, y) \in L_w, \\ 0, & (x, y) \notin L_o \cup L_w, \end{cases} \quad (19)$$

$$c_{pw} = \begin{cases} c_p, & (x, y) \in L_o, \\ c_{pw}, & (x, y) \in L_w, \\ 0, & (x, y) \notin L_o \cup L_w, \end{cases} \quad (20)$$

where the water cut of production well $f_w$ is given by

$$f_w = \frac{1}{1 + \frac{c_{pw}}{c_{pw}}}. \quad (21)$$

The control for the process is the polymer concentration of the injected fluid $c_{\text{pin}}$ denoted as

$$c_{\text{pin}}(x_i, y_j, t_k) = \begin{cases} v_i(k), & t \in [t_{k-1}, t_k], \quad k = 1, 2, \ldots, P - 1, \\ 0, & t \in [t_{P-1}, t_f], \end{cases} \quad (22)$$

where $i = 1, 2, \ldots, N_w$ denotes the $i$th injection well, $v_i(k)$ the polymer concentration for the $k$th slug of the $i$th injection well, $P - 1$ the number of polymer slugs and $t_k - t_{k-1}$ the integer time stage length in days of the $k$th slug. The slug switching time $t_k$ for every injection well is assumed consistent in this article. The last time stage $P$ is the water flooding process after polymer injection, where the injection polymer concentration is $c_{\text{pin}} \equiv 0$ g/L.

The injection scheme design of polymer flooding is to determine the volume size, the injection polymer concentration of every slug and the free terminal time. Because the injection velocity $q_{\text{in}}$ is constant here, the slug size is determined by the time stage length $t_k - t_{k-1}$. The terminal time $t_f$ is determined by the sum of the time stage lengths of every polymer slug and water flooding after polymer injection. Therefore, the optimisation variables include the injection polymer concentration $v_i$, $v_i(0) = 0$ and the time stage length $t_k - t_{k-1}, k = 1, \ldots, P$ ($t_P = t_f$) of every slug. Note that $v_i(k)$ is a real variable and $t_k - t_{k-1}$ is an integer variable in days. So, the injection scheme design of polymer flooding is a mixed-integer OCP governed by Equations (4)–(8). Figure 1 shows a three-slug injection scheme of polymer flooding.

### 2.3 Constraints

The constraints in polymer flooding are expressed mathematically as follows:

- The polymer injection amount constraint
  $$\int_0^{t_f} \int_\Omega q_{\text{in}} c_{\text{pin}} \, d\sigma \, dt \leq m_{p_{\text{max}}}. \quad (23)$$

- The injection polymer concentration constraint
  $$0 \leq c_{\text{pin}} \leq c_{\text{max}}. \quad (24)$$

- The terminal state constraint
  $$f_w|_{t=t_f} = 98\%, \quad (25)$$

where $m_{p_{\text{max}}}$ is the maximum polymer injection amount and $c_{\text{max}}$ the maximum injection polymer concentration. Equation (25) means that the oil displacement process is terminated when the water cut of production well reaches 98%.

### 3. Full implicit finite-difference method

The governing equations given by Equations (4)–(8) are nonlinear PDEs. Several finite-difference approximations for the numerical simulation of such DPS are possible. We adopt a full implicit finite-difference scheme for the calculation of the governing equations. The scheme is described below.

For two space variables, we now consider the grid system with which we divide up the reservoir region in the $x - y$ plane. The integer $i$ ($i = 1, 2, \ldots, n_x$) is used as the index in the $x$-direction and the integer $j$ ($j = 1, 2, \ldots, n_y$) for the index in the $y$-direction. Thus, $x_i$ is the $i$th value of $x$ and $y_j$ is the $j$th value of $y$. Double indexing is used to identify functions within the two-dimensional region. Let $u(x, y, t) = (p, S_w, c_p)^T$
denote the system state vector. The state vector in grid point \((x_i, y_j)\) is described by
\[ u_{ij} = u(x_i, y_j). \] (26)

The reservoir domain is divided into \(n_x \times n_y\) blocks, and the point \((x_i, y_j)\) is considered to be at the centre of block \((i, j)\). There are \(n_x\) blocks in the \(x\)-direction and \(n_y\) blocks in the \(y\)-direction. Further details concerning this grid are given in Figure 2. We identify the coordinate \(x_{i-\frac{1}{2}}\) with the left side of the block \((i, j)\) and \(x_{i+\frac{1}{2}}\) with the right side of the block. Similarly, \(y_{j-\frac{1}{2}}\) is identified with the bottom of the block and \(y_{j+\frac{1}{2}}\) with the top.

Using the predefined grid system, the derivatives in Equations (4)–(6) are replaced by finite differences. Three formulae are useful in this context:
\[
\begin{align*}
\frac{\partial u}{\partial t} &= \frac{u^{n+1} - u^n}{\Delta t}, \quad (27) \\
\frac{\partial u}{\partial x} &= \frac{u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j}}{\Delta x}, \quad (28) \\
\frac{\partial u}{\partial y} &= \frac{u_{i,j+\frac{1}{2}} - u_{i,j-\frac{1}{2}}}{\Delta y}, \quad (29)
\end{align*}
\]
where \(n\) is the index of reservoir simulation time step, \(n = 0, 1, \ldots, N - 1\), \(N\) the number of simulation time steps, \(\Delta t\) the size of the \(n\)th time step in days, \(u^{n+1}\) the state vector at time \(t^{n+1}\), \(t^n\) the total simulation time in days at the end of the \(n\)th time step and \(\Delta x\) and \(\Delta y\) \((m)\) the space lengths in the \(x\)- and \(y\)-directions, respectively.

Apply the formulae (27)–(29) to discretise the governing equations (4)–(6), and multiply the grid area \(\Delta x \Delta y \left( m^2 \right)\) on both sides of the equations. For the

full implicit finite-difference method, the second spatial derivative items in Equations (4)–(6) are evaluated at time \(t^{n+1}\) instead of at \(t^n\). Then, the full implicit finite-difference equations for the grid point \((i, j)\) at time \(t^n\) are given by
\[
\begin{align*}
\frac{h\Delta y}{\Delta x} \left[ \frac{r_{ij}^{n+1}}{C_0} (p_{i+1,j}^{n+1} - p_{ij}^{n+1}) + \frac{r_{ij}^{n+1}}{C_0} (p_{i,j+1}^{n+1} - p_{ij}^{n+1}) \right] &= Q_o \\
\frac{h\Delta x}{\Delta y} \left[ \frac{r_{ij}^{n+1}}{C_0} (p_{i,j}^{n+1} - p_{ij}^{n+1}) + \frac{r_{ij}^{n+1}}{C_0} (p_{ij}^{n+1} - p_{i-1,j}^{n+1}) \right] - Q_w \\
&= V \left[ \frac{\phi_{ij}^{n+1} (1 - S_{w_{nlj}}^{n+1})}{B_{nlj}^{n+1}} - \frac{\phi_{ij}^n (1 - S_{w_{nlj}}^n)}{B_{nlj}^n} \right], \quad (30)
\end{align*}
\]
\[
\begin{align*}
\frac{h\Delta y}{\Delta x} \left[ \frac{D_{ij}^{n+1}}{C_0} (\rho_{ij}^{n+1} - \rho_{ij}^n) + \frac{D_{ij}^{n+1}}{C_0} (\rho_{ij}^{n+1} - \rho_{ij}^{n-1}) \right] &= Q_u c_p \\
\frac{h\Delta x}{\Delta y} \left[ \frac{D_{ij}^{n+1}}{C_0} (\rho_{ij}^{n+1} - \rho_{ij}^n) + \frac{D_{ij}^{n+1}}{C_0} (\rho_{ij}^{n+1} - \rho_{ij}^{n-1}) \right] - Q_w \\
&= V \left[ \frac{\phi_{ij}^{n+1} S_{nlj}^{n+1}}{B_{nlj}^{n+1}} - \frac{\phi_{ij}^n S_{nlj}^n}{B_{nlj}^n} \right], \quad (31)
\end{align*}
\]
\[
\begin{align*}
\frac{h\Delta y}{\Delta x} \left[ \frac{r_{ij}^{n+1}}{C_0} (p_{i+1,j}^{n+1} - p_{ij}^{n+1}) + \frac{r_{ij}^{n+1}}{C_0} (p_{i,j+1}^{n+1} - p_{ij}^{n+1}) \right] &= Q_o \\
\frac{h\Delta x}{\Delta y} \left[ \frac{r_{ij}^{n+1}}{C_0} (p_{i,j}^{n+1} - p_{ij}^{n+1}) + \frac{r_{ij}^{n+1}}{C_0} (p_{ij}^{n+1} - p_{i-1,j}^{n+1}) \right] - Q_w \\
&= V \left[ \frac{\phi_{ij}^{n+1} (1 - S_{w_{nlj}}^{n+1})}{B_{nlj}^{n+1}} - \frac{\phi_{ij}^n (1 - S_{w_{nlj}}^n)}{B_{nlj}^n} \right], \quad (32)
\end{align*}
\]
where $Q_o$ is the source term of the discretised oil phase flow equation, $(Q_o = \Delta x \Delta y q_o, \text{m}^3/\text{day})$, $Q_w$, the source term of the discretised water phase flow equation, $(Q_w = \Delta x \Delta y q_w, \text{m}^3/\text{day})$, $V$ the grid volume $(V = h \Delta x \Delta y, \text{m}^3)$ and the corresponding parameters are $r_p = \frac{\mu_p}{\mu_w}$, $r_w = \frac{\mu_w}{\mu_o}$, $r_d = \frac{\phi}{\mu_o}$ and $r_c = \frac{\phi}{\mu_o}$.

We adopt the two-point upstream weighting method (Aziz and Settari 1986) to deal with the parameter value at the side of the block. Apply Equations (30)–(32) at every grid point in the reservoir region, then we can obtain $n_x \times n_y \times 3$ nonlinear algebraic equations with respect to the discrete states $u^{i,j}_{n+1}$, $i = 1, \ldots, n_x$, $j = 1, \ldots, n_y$. The usual method to solve the algebraic equations is through the Newton–Raphson algorithm. Details of the algorithm can be found in Aziz and Settari (1986).

The boundary condition (7) is equivalent to
\[ \frac{\partial u}{\partial x} = 0, \quad \frac{\partial u}{\partial y} = 0, \quad \forall (x, y, t) \in \Omega \times [0, t_f]. \] (33)

Therefore, using finite difference and Equation (33), we have
\[ u_{i,0} = u_{i,1}, \quad u_{i,n_y+1} = u_{i,n_y}, \quad i = 1, 2, \ldots, n_x, \] (34)
\[ u_{0,j} = u_{1,j}, \quad u_{n_x+1,j} = u_{n_x,j}, \quad j = 1, 2, \ldots, n_y. \] (35)

On solving the governing equations (4)–(8) by the full implicit finite-difference methods, a problem that was originally described by PDEs defined over continuous time and spatial domains is transformed to problem that is described by a set of discrete algebraic equations that are implicit form.

4. Mixed-integer IDP

We consider the discrete DPS described for system (4)–(8). For maximisation of the performance index (3), it should allow the discussion of the optimisation procedure. The initial ideas on IDP were developed and tested by Luus (1990a) and then refined (Luus 1989) to make the computational procedure much more efficient. More information can find in Luus (2000).

4.1 Mixed-integer formulation for IDP

Since IDP is a stagewise optimisation procedure where the time stage lengths are determined by the terminal time (Luus 1990b), we normalise the time variable in the OCP of polymer flooding, so that the time stages that are of varying lengths in the original problem become stages of constant length in the normalised time. This normalisation allows us to determine simultaneously the optimal injection polymer concentration and the time stage length of every slug, as well as the terminal time.

The optimisation of continuous injection polymer concentration is transformed into a piecewise-constant control problem by the slug injection pattern, as shown in Equation (22). Then, the time interval $0 \leq t < t_f$ is divided into $P$ time stages with each of variable integer length given by
\[ \bar{s}(k) = t_k - t_{k-1}, \quad k = 1, 2, \ldots, P, \] (36)
where the stage length $\bar{s}(k)$ is an integer value in days and
\[ \bar{s}(k) \geq 0, \quad k = 1, 2, \ldots, P. \] (37)

The polymer injection concentration is kept constant during each of these time intervals. The performance index (3) is discretised as
\[ \max J = \xi_s \sum_{n=0}^{N-1} \left[ \Delta t^n \sum_{j=1}^{N_y} \left( 1 - \frac{f_{w,j}}{s_{\text{w}}^*} \right) Q_{\text{out}} \right] \] (38)
\[ - \xi_p \sum_{k=1}^{P-1} \left[ \bar{s}(k) \sum_{j=1}^{N_y} v_j(k) Q_{\text{in}} \right], \]
where $Q_{\text{out}}$ is the fluid production rate $(Q_{\text{out}} = \Delta x \Delta y q_{\text{out}}, \text{m}^3/\text{day})$, $Q_{\text{in}}$ the fluid injection rate $(Q_{\text{in}} = \Delta x \Delta y q_{\text{in}}, \text{m}^3/\text{day})$ and $f_{w,j}$ the water cut of the $j$th production well.

Equations (4)–(6) can be described as the general form:
\[ \frac{\partial g(u)}{\partial t} = f(u, u_x, u_y, u_{xx}, u_{yy}, v), \] (39)
where $v(x, y, t), (x, y) \in L_w$ denotes the continuous injection polymer concentration vector, $u = \frac{\phi}{\mu_o}$, $u_x = \frac{\phi}{\mu_o}$, $u_{xx} = \frac{\phi}{\mu_o}$ and $u_{yy} = \frac{\phi}{\mu_o}$.

Let us introduce a normalised time variable $\tau$, so that the time stages are of equal length, by defining $d\tau$ in the time interval $t_{k-1} \leq t \leq t_k$ through the relationship
\[ d\tau = \bar{s}(k)^P d\tau. \] (40)

By integrating Equation (40) over a stage, we get
\[ t_k - t_{k-1} = \bar{s}(k) P (\tau_k - \tau_{k-1}), \] (41)
so that the length of each time stage in normalised time is
\[ \tau_k - \tau_{k-1} = \frac{1}{P}. \] (42)
Since \( \tau_0 = 0 \), the normalised slug switching time is

\[ \tau_k = k/P, \quad k = 1, 2, \ldots, P. \]  

(43)

The governing equations thus becomes

\[ \frac{\partial g(u)}{\partial \tau} = \tilde{s}(k) P f(u, u_x, u_y, u_{xx}, u_{yy}, v), \quad k = 1, 2, \ldots, P \]  

(44)

in the time interval \( \tau_{k-1} \leq \tau \leq \tau_k \). With this transformation, at \( t = \tau_k \), \( \tau_P = 1 \), and in the dimensionless time domain, all stages are of equal length. The OCP is to determine the real injection polymer concentration \( v(k) = [v_1(k), v_2(k), \ldots, v_N(k)] \) and also the integer time stages length \( \tilde{s}(k), k = 1, 2, \ldots, P \), such that the performance index (38) governed by Equation (44) is maximised. As formulated, the problem is now well-suited for IDP. But the traditional IDP is not adapted for mixed-integer OCP. A mixed-integer IDP is proposed here to optimise the real and integer variables simultaneously.

4.2 Algorithm implementation

The proposed mixed-integer IDP with a truncation procedure for integer restrictions on stage lengths for the OCP of polymer flooding can be summarised in seven steps.

**Step 1:** Initialisation: The process of polymer flooding is divided into \( P \) time stages, each of normalised length \( 1/P \). Choose the initial values for \( v^{(0)}(k) \) and \( \tilde{s}^{(0)}(k), k = 1, 2, \ldots, P \) for each stage, and the initial region size \( r^{(0)}(k) \) for each of the injection polymer concentration \( v^{(0)}(k) \) and the initial region \( q^{(0)}(k) \) for the time stage length \( \tilde{s}^{(0)}(k) \). Note that the injection polymer concentration at the last stage is \( v(P) \equiv 0 \). Select the number of allowable values for control \( R > 1 \) to be tried at each stage \( k = 1, 2, \ldots, P \). Choose the region contraction factor \( 0.5 < \gamma < 1.0 \) and the number of grid points \( M \) for the states. Finally, specify the number of iterations \( I_{\text{max}} \). Set iteration number \( I = 1 \).

**Step 2:** Use the best control policy from the previous iteration (the guess solution at iteration 1) \( v^{(I-1)}(k) \) and \( \tilde{s}^{(I-1)}(k) \) for each stage, and generate \( M-1 \) other injection polymer concentrations within the region \( v^{(I-1)}(k) \pm \gamma^{(I-1)}(k) \) and integer time stages within the region \( \tilde{s}^{(I-1)}(k) \pm q^{(I-1)}(k) \). Solve the governing equations (Equation (44)) by the full implicit finite-difference method in Section 3 from \( \tau = 0 \) to \( \tau = 1 \) for all \( M \) control policies. The \( M \) values of the discrete states \( u_{ij}, i = 1, \ldots, n_x, j = 1, \ldots, n_y \), at the beginning of each time stage constitute the \( M \) grid points at each stage.

**Step 3:**

1. At stage \( P \), for each of the \( M \) stored values for \( u_{ij}(\tau_{P-1}), i = 1, \ldots, n_x, j = 1, \ldots, n_y \), solve the dynamic equation (Equation (44)) by the full implicit finite-difference method from the normalised time \( \tau_{P-1} = (P-1)/P \) to \( \tau_P = 1 \), with the injection polymer concentration \( v(P) \equiv 0 \) and each of the \( R \) allowable values for the stage length \( \tilde{s}(P) \), which are generated by:

\[ s^{(I)}(P) = \tilde{s}^{(I-1)}(P) + w q^{(I-1)}(P), \]  

(45)

where \( \tilde{s}^{(I-1)}(P) \) is the best stage length obtained in the previous iteration and \( w \) a random number between \(-1\) and 1. In order to ensure that the integer restrictions for the stage lengths are satisfied, the following truncation procedure is applied. The value of \( s^{(I)}(P) \) obtained from Equation (45) is truncated to integer value \( \hat{s}^{(I)}(P) \) by the rule:

- if \( s^{(I)}(P) \) is integer, then \( \hat{s}^{(I)}(P) = s^{(I)}(P) \), otherwise

- \( \hat{s}^{(I)}(P) \) is equal to either \( [s^{(I)}(P)] \) or \( [s^{(I)}(P)] + 1 \) each with probability 0.5, \( ([s^{(I)}(P)] \) is the integer part of \( s^{(I)}(P) \).

This ensures greater randomness in the set of time stage lengths being generated and avoids the possibility of the same integer values being generated whenever a real value lying between the same two consecutive integers is truncated. The stage length \( \hat{s}^{(I)}(P) \) is a semi-definite integer variable, as specified in Equation (37).

2. For each grid point, we therefore have \( R \) values of performance index to compare and we choose the stage length \( \tilde{s}^{(I)}(P) \) that gives a maximum for \( J \) in Equation (38). The best injection polymer concentration at the last stage is considered as \( v^{(I)}(P) = 0 \). The corresponding injection concentration and stage length are stored for use in Step 4.

**Step 4:**

1. Step back to stage \( P-1 \), corresponding to the normalised switching time \( \tau_{P-2} = P-2/P \), and for each of the \( M \) state grid points \( u_{ij}(\tau_{P-1}), i = 1, \ldots, n_x, j = 1, \ldots, n_y \), generate \( R \) sets of allowable values for the stage length

\[ s^{(I)}(P-1) = \tilde{s}^{(I-1)}(P-1) + w q^{(I-1)}(P-1), \]  

(46)

and for the injection polymer concentration

\[ v^{(I)}(P-1) = v^{(I-1)}(P-1) + D r^{(I-1)}(P-1), \]  

(47)
where \( D \) is an \((N_w \times N_w)\) diagonal matrix with different random numbers between \(-1\) and \(1\) along the diagonal and \( v^{(l)}(P - 1) \) the best injection polymer concentration obtained in the previous iteration. The real value \( s^{(l)}(P - 1) \) is truncated to integer time stage length \( \tilde{s}^{(l)}(P - 1) \) by the truncation procedure in Step 3.

To deal with constraints of the injection polymer concentration (Equation (24)) whenever an unfeasible solution is generated, it is set to the violated limit, according to:

\[
v^{(l)}_i(P - 1) = \begin{cases} 
0, & \text{if } v^{(l)}_i(P - 1) < 0, \\
\epsilon_{\text{max}}, & \text{if } v^{(l)}_i(P - 1) > \epsilon_{\text{max}}, 
\end{cases} \quad i = 1, 2, \ldots, N_w.
\]

(48)

Now integrate Equation (44) from \( \tau_{P-2} \) to \( \tau_{P-1} \) with each of the \( R \) sets of allowable values.

(2) The full implicit finite difference is continued from \( \tau_{P-1} \) to \( \tau_P \) using the stored value for \( v^{(l)}(P) \) and \( \tilde{s}^{(l)}(P) \) that corresponds to the grid point closest to the value of the discrete states \( u_{i,j}, \quad i = 1, \ldots, n_x, \quad j = 1, \ldots, n_y, \) at time \( \tau_{P-1} \). Compare the \( R \) values of the performance index \( J \) and store the injection concentration \( v^{(l)}(P - 1) \) and stage length \( \tilde{s}^{(l)}(P - 1) \) that give the best performance over the normalised time interval \([\tau_{P-2}, \tau_P]\).

Step 5: Repeat Step 4 until stage 1, corresponding to the initial time \( \tau = 0 \), is reached. Here, there is only one state grid point that corresponds to the initial conditions (Equation (35)). Store the injection polymer concentration \( v^{(l)}(k) \) and the stage length \( \tilde{s}^{(l)}(k) \) of each stage that yield the maximum value for the performance index \( J \) in Equation (38).

Step 6: Reduce the region for allowable values for the injection polymer concentration

\[
v^{(l+1)}(k) = \gamma v^{(l)}(k), \quad k = 1, 2, \ldots, P,
\]

and the stage length

\[
v^{(l+1)}(k) = \gamma q^{(l)}(k), \quad k = 1, 2, \ldots, P.
\]

(50)

Select the best injection polymer concentration and the stage lengths from Step 5 as the midpoint for the next iteration.

Step 7: Set \( l = l + 1 \) and go to Step 2. Continue the procedure for the specified number of iterations \( l_{\text{max}} \).

For more information about that we are able to use the polymer injection amount constraint (23) and the terminal state constraint (25) within IDP method, see Luus (2000, 2009).

5. Numerical example

In this section, we present a numerical example of optimal control for polymer flooding done with the mixed-integer IDP method. Herein, we also present comparisons with the traditional trial and error method.

The two-phase flow of oil and water in a heterogeneous two-dimensional reservoir with 90 \((9 \times 10 \times 1)\) grid blocks is considered. The dimensions of the reservoir were 421.02 \(\times\) 443.8 \(\times\) 5 m\(^3\). The production model is a five-spot pattern, with one production well located at the centre of the reservoir \( (5, 6) \) and four injection wells placed at the four corners \((1, 10), (9, 10), (1, 1), (9, 1)\), and \((9, 1)\), as shown in the permeability distribution map of Figure 3. Polymer is injected when the water cut of the production well comes to 97% after water flooding. Figures 4 and 5 show the contour maps of the initial water saturation \( S_w^0 \) and the initial reservoir pressure \( p^0 \), respectively. The initial polymer concentration is \( \epsilon_p^0 = 0 \) (g/L). A common three-slug injection scheme is used in this example, so that the polymer flooding process is divided into four stages \((P = 4)\). There are 16 control variables for optimisation including 3 polymer injection concentrations of each production well and 4 time stage lengths of all production wells. In the performance index calculation, we use the price of oil \( \xi_o = 0.0503 (10^4 \text{$/m}^3)\) [80 ($/bbl)] and the cost of polymer \( \xi_p = 2.5 \times 10^{-4} (10^4 \text{$/kg})\). The fluid rate of the production well is \( Q_{\text{out}} = 60 \text{m}^3/\text{day} \) and the fluid rate of every injection well is \( Q_{\text{in}} = 15 \text{m}^3/\text{day} \). For the constraint (24), the maximum injection polymer concentration is \( \epsilon_{\text{max}} = 2.5 \) (g/L). The parameters of the reservoir include...
The injection polymer concentration and stage length of every polymer slug obtained by the conventional trial and error method are
\[ v_i^k(k) = 1.8 \text{ (g/L)}, \quad i = 1, 2, \ldots, N_w, \quad \text{and} \quad s^k(k) = 500 \text{ (days)}, \quad k = 1, 2, 3. \]

When the water cut of the production well reaches 98\%, the terminal time \( t_f = 5390 \text{ (days)}. \)

The performance index is \( J^* = 1.618 \times 10^7 \) with oil production 32,167 m\(^3\) and polymer injection 162,000 kg.

For comparison, the results obtained by the trial and error method are considered as the initial values of the proposed mixed-integer IDP. The maximum polymer injection amount is \( m_{p_{\text{max}}} = 162,000 \text{ (kg)}. \)

The parameters of the proposed algorithm for this example are set as follows: the number of states grid points is \( M = 3 \), the number of allowable values for control is \( R = 15 \), the region contraction factor is \( \gamma = 0.85 \), the initial region size is \( r^0(k) = 1 \text{ (g/L)} \), the initial region is \( q^0(k) = 100 \text{ (days)} \) and the number of iterations \( l_{\text{max}} = 30 \).

Using the mixed-integer IDP algorithm, we obtain a cumulative oil of 32,660 m\(^3\) and a cumulative polymer of 162,000.01 kg, yielding the profit of \( J^* = 1.643 \times 10^7 \) over the polymer flooding project life of the reservoir. The results show an increase in performance index of \( 2.5 \times 10^5 \). The optimised terminal time is \( t_f = 5162 \text{ (days)}. \)

Figures 6 and 7 show the water cut curves and the cumulative oil production of the two methods, respectively. It is obvious that the water cut of mixed-integer IDP is lower than that of trial and error. Therefore, the proposed method gets more oil production and higher recovery ratio. Figures 8–11 show the optimal control strategies of the injection wells W1–W4. As a result, the optimal injection polymer concentration profiles of W1 and W2 are significantly different from those of W3 and W4. It is mainly due to the differences of the well positions and the distance to the production well, as well as the reservoir heterogeneity and the uniform initial water saturation distribution.

6. Conclusions

In this study, a mixed-integer IDP method is first proposed to numerically solve the OCP of DPS for polymer flooding in EOR. The main conclusions obtained from this study are as follows.

A new mixed-integer optimal control model is established for the dynamic injection strategies leading to polymer flooding. In this model, the injection polymer concentration, the slug size and the terminal time of polymer flooding are simultaneously considered as the optimisation variables, so that the practical engineering requirements can be satisfied.

A new mixed-integer optimal control model is established for the dynamic injection strategies leading to polymer flooding. In this model, the injection polymer concentration, the slug size and the terminal time of polymer flooding are simultaneously considered as the optimisation variables, so that the practical engineering requirements can be satisfied.

A new mixed-integer IDP algorithm incorporating a special truncation procedure for integer stage length restriction is developed to solve the OCP of the DPS. The mixed-integer IDP turns out to be a labour-saving tool independent on the intuition of engineers. It can effectively reduce the number of man-hours needed to
Table 1. Parameters of reservoir description used in the example.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of production well, ( N_p )</td>
<td>1</td>
</tr>
<tr>
<td>Number of injection wells, ( N_w )</td>
<td>4</td>
</tr>
<tr>
<td>Number of grids in ( x )-direction, ( n_x )</td>
<td>9</td>
</tr>
<tr>
<td>Number of grids in ( y )-direction, ( n_y )</td>
<td>10</td>
</tr>
<tr>
<td>Space step length in ( x )-direction, ( \Delta X ) (m)</td>
<td>46.78</td>
</tr>
<tr>
<td>Space step length in ( y )-direction, ( \Delta Y ) (m)</td>
<td>44.38</td>
</tr>
<tr>
<td>Thickness of the reservoir bed, ( h ) (m)</td>
<td>5</td>
</tr>
<tr>
<td>Reference pressure, ( p_r ) (MPa)</td>
<td>12</td>
</tr>
<tr>
<td>Porosity under the condition of the reference pressure, ( \phi_r )</td>
<td>0.31</td>
</tr>
<tr>
<td>Rock density, ( \rho ) (kg/m(^3))</td>
<td>2000</td>
</tr>
<tr>
<td>Rock compressibility factor, ( C_R ) (1/MPa)</td>
<td>( 9.38 \times 10^{-6} )</td>
</tr>
<tr>
<td>Irreducible water saturation, ( S_{ir} )</td>
<td>0.25</td>
</tr>
<tr>
<td>Residual oil saturation, ( S_{wo} )</td>
<td>0.22</td>
</tr>
<tr>
<td>Oil relative permeability at the irreducible water saturation, ( k_{oro} )</td>
<td>0.5228</td>
</tr>
<tr>
<td>Water relative permeability at the residual oil saturation, ( k_{wrcw} )</td>
<td>0.9</td>
</tr>
<tr>
<td>Index of oil relative permeability curve, ( n_o )</td>
<td>4.287</td>
</tr>
<tr>
<td>Index of water relative permeability curve, ( n_w )</td>
<td>2.3447</td>
</tr>
</tbody>
</table>

Table 2. Fluid data used in the example.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil viscosity, ( \mu_o ) (mPa · s)</td>
<td>50</td>
</tr>
<tr>
<td>Compressibility factors of oil, ( C_o ) (1/MPa)</td>
<td>( 5 \times 10^{-6} )</td>
</tr>
<tr>
<td>Oil volume factor under the condition of the reference pressure, ( B_o )</td>
<td>1</td>
</tr>
<tr>
<td>Aqueous phase viscosity with no polymer, ( \mu_w ) (mPa · s)</td>
<td>0.458</td>
</tr>
<tr>
<td>Compressibility factors of water, ( C_w ) (1/MPa)</td>
<td>( 4.6 \times 10^{-6} )</td>
</tr>
<tr>
<td>Water volume factor under the condition of the reference pressure, ( B_w )</td>
<td>1</td>
</tr>
<tr>
<td>Polymer absorption parameter, ( a ) (g/cm(^3))</td>
<td>0.03</td>
</tr>
<tr>
<td>Polymer absorption parameter, ( b ) (g/cm(^3))</td>
<td>3.8</td>
</tr>
<tr>
<td>Diffusion coefficient, ( D ) (m(^2)/s)</td>
<td>( 1 \times 10^{-5} )</td>
</tr>
<tr>
<td>Permeability reduction parameter, ( R_{kmax} )</td>
<td>1.15</td>
</tr>
<tr>
<td>Permeability reduction parameter, ( b_{rk} )</td>
<td>1.2</td>
</tr>
<tr>
<td>Viscosity parameter, ( a p_1 )</td>
<td>15.426</td>
</tr>
<tr>
<td>Viscosity parameter, ( a p_2 )</td>
<td>0.4228</td>
</tr>
<tr>
<td>Viscosity parameter, ( a p_3 )</td>
<td>0.2749</td>
</tr>
</tbody>
</table>

Figure 6. Water cut of the production well P1.

Figure 7. Cumulative oil production.
determine the best injection policy. Furthermore, different from the method of maximum principle, the proposed methodology can avoid solving the complex adjoint equations.

The optimal control model of polymer flooding and the mixed-integer IDP are used for a reservoir example and the optimum injection concentration profiles for each well are offered. The results show that the profit is enhanced by the proposed method. Meanwhile, more oil production and higher recovery ratio are obtained. And the injection strategies chosen by trial and error are same for all the wells, whereas the optimal control policies by the proposed method are different from each other as a result of the reservoir heterogeneity and the uniform initial conditions.

In conclusion, given the properties of an oil reservoir and the properties of a polymer solution, an optimal polymer flooding injection policy to maximise profit can be designed by using the proposed methodology. The approach used is a powerful tool that can aid significantly in the development of operational strategies for EOR processes.

Acknowledgements

This study was supported by the ‘Natural Science Foundation of China’ under Grant 60974039, the ‘China Important National Science Technology Specific Projects’ under Grant 2008ZX05011, the ‘Natural Science Foundation of Shandong Province of China’ under Grant ZR2011FM002 and the ‘Independent Innovation Scientific Research Program of China University of Petroleum (East China)’. The authors are grateful to the Research Institute of Geological Science of Sinopec Shengli Oilfield Company for its technical assistance.
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