Polychromatic set theory-based spectrum access in cognitive radios

S. Li X. Wang

Abstract: In this study, the authors have investigated a dynamic spectrum access method for cognitive radio (CR) networks by using polychromatic sets (PS) theory. First, a power control model is proposed in which the transmission power at a CR node can be calculated by considering both the primary radio (PR)-to-CR and PR-to-PR interference under a specific outage probability. In order to allocate the available spectrum among the CR nodes in a spectrum overlay scenario, the authors further propose a channel selection algorithm based on PS. This study also gives a framework of the PS-based method and concludes with future work describing the practical implementation of the proposed framework. Effectiveness of proposed method is demonstrated through an example application.

1 Introduction

Recently, cognitive radio networks (CRNs) have received a large amount of interest as a novel means to improve spectrum utilisation [1–3]. The concept of cognitive radio (CR) was first proposed by Mitola and Maguire in [4] as a promise to improve the spectrum utilisation and sharing among licensed and unlicensed users. In solutions for CRNs, spectrum access would make it the coexistence between the primary radio (PR) users and CR users possible [5, 6]. In a CRN, three major dynamic spectrum access models can be identified: the common used model, the shared-use model of the PR spectrum and the exclusive-use model [5]. Actually, many challenges still need to be solved in designing an efficient resource management scheme for dynamic and autonomous applications over CRNs [5–7]: (i) Cognitive capability. In CRNs, CR nodes should be able to sense information from their surroundings in order to figure out spectrum portions that are unused at a specific time or location. The most suitable portion would be selected for communications without causing interference to PR users. More precisely, a cognitive CR node should have the capability of spectrum sensing, spectrum access, communication and spectrum mobility. (ii) Reconfigurability. CR nodes should be able to dynamically reprogram according to the requirement of the real environment.

Numerous efforts have been focused on dynamic spectrum access in CRNs; however, there are still many challenges for designing efficient resource management solutions for dynamic and autonomous applications over CRNs [6, 7]: (i) The dynamic, time-varying nature of resource, applications and channel characteristics influence the performance of different transmission strategies at various layers and the choice of optimal strategy adopted by the CR transmitter. (ii) The transmission and resource strategies significantly affect the action and performance of a CRN. Thus, the interactions between CRs and PRs are very important for dynamic spectrum access. (iii) Cooperation between CRs and PRs. Existing schemes always neglect how users acquire information [7, 8], how they interact and compete with each other in repeated or stochastic games based on their local information [6], and how they learn over time based on acquired information. Actually, these questions should be taken into account cooperatively in designing a CRN solution.

For a successful operation of a CRN node, three essential problems need to be addressed: (i) discovery of available channel opportunities by CR nodes, (ii) improving the utilisation of available channel opportunities and (iii) routing scheme based on the available channels for the CRN. For the first problem, there have been substantial research efforts on spectrum sensing and detecting. However, the second problems is still an open issue. There is limited amount of work to date available in the literature for dynamic spectrum assignment problem in CRN. In [2], Khalife et al. described the problem as a selection problem based on probabilistically estimating the available capacity of every channel. However, because the dynamic channel selection depends on a great extent on the interference caused by CR-to-PR, and thus this is not applicable to power/rate control problems [2]. In [9–12], Hou and Shi studied a formal mathematical model for joint power control, scheduling and routing. Hou and Shi considered spectrum sensing and decision-making process from a cross-layer optimisation viewpoint, with joint consideration of power control, scheduling and routing. The first two problems can be simplified by restricting the treatment to CR nodes only. So the PR-to-PR and PR-to-CR interferences do not appear in their formulation. For the third problem, the authors in [12] introduced the routing metrics in
CRNs. The works in [1, 13–16] introduced several different routing schemes for CRNs. Wu and Tsang [14] used the concept of time-spectrum blocks to study the spectrum allocation in CRNs. Actually, many works research for spectrum allocation in CRNs have been done based on a continuous-time Markov model, such as in [17, 18]. In [17], Xing et al. proposed a random access protocol that achieved airtime fairness among CRs. The works in [19–21] compared the existing spectrum access methods for CRNs.

In this paper, we propose a mathematical framework by using polychromatic sets theory (PST) that enables us to design, analyse and optimise dynamic CR environments and applications flexibly. The main contributions of this paper are as follows: (i) We formulated the channel access as a generalised multi-level spectrum opportunity framework, which takes into consideration the inference that is caused by other CRs and PRs. (ii) We proposed an effective and adaptive spectrum sensing scheme and available channels calculation algorithm based on our previous work on PST. (iii) Proposed spectrum access scheme is able to couple the internal and external actions of the CR nodes to allow them to achieve an optimal response to the dynamically changing wireless resource markets.

2 System model

2.1 System model

In this section, we consider a spectrum-leasing scenario in which CRN is a sharing spectrum with an infrastructured primary radio network (PRN). We assume that the PRN consists of multiple static PR nodes being interconnected via a fixed extra-channel and cannot be disturbed by CR or other PRNs. Each PR periodically reports its status on each channel to other PRs, and the collected real-time channel status information is available for CRN, which can be built in a polychromatic matrix conveniently. Each CR is able to calculate the power mask to adapt its interference range to the activity of neighbouring PR nodes. By doing this, the CR can calculate its spectrum opportunity. The interference range can be obtained according to the propagation distance between nodes.

We consider a CRN model with \( N \) nodes. For each CR there is a set of available frequency bands that are different depending on the location and working status of CR and its neighbouring PRs. Let \( M \) denote all channels in the CRN and \( M_i \) denote the set of available frequency bands at node CR \( i \), we have

\[
M = \bigcup_{i=1}^{N} M_i \tag{1}
\]

For each channel at node \( i \), we assume that the bandwidth is \( B_i \). In this paper, we use a Rayleigh process to model the fading of a channel between two nodes \( i \) and \( j \). The received signal power at \( j \) over channel \( m \) can be calculated by (2) [22]

\[
P_{ij}^{(m)} = A_0^m \left( \frac{d_j}{d_{ij}^{(m)}} \right)^{-n} P_{ij}^{(m)}, \quad d \geq d_{ij}^{(0)} \tag{2}
\]

where \( A_0^m = P_i^{(m)} G_i^{(m)} G_r^{(m)} (4\pi d_{ij}^{(m)})^{-2} \) is the path loss in which \( d_{ij}^{(0)} \) is the reference distance, \( P_i^{(m)} \) is the transmission power, \( G_i^{(m)} \) is the antenna gain of the transmitter, \( G_r^{(m)} \) is the antenna gain of the receiver, \( \lambda \) is the wavelength and \( P_{ij}^{(m)} \) is a normalised random variable that represents the power gain of the fading process.

Consider a transmission from node \( i \) to \( j \) over channel \( m \), there is a limitation on transmission power for a concurrent transmission from a neighbouring node \( k \) to another node \( h \). In order to ensure the successful transmission over channel \( m \), assume that the received interference power at node \( j \) can be negligible if it does not exceed a threshold \( P_T \), that is, \( \sum_{k \neq i} P_k^{(m)} G_{kj}^{(m)} \leq P_T \). So, we can calculate the maximum allowed transmission power at node \( k \) when node \( i \) is transmitting as follows

\[
\sum_{k \neq i} P_k^{(m)} G_{kj}^{(m)} < P_T \tag{3}
\]

For node \( i \), if the data can be successfully transmitted to node \( j \), the received power at \( j \) should exceed a power threshold \( P_T \), that is, \( P_i^{(m)} G_{ij}^{(m)} \geq P_T \). So, the minimum required transmission power at node \( i \) is

\[
P_i^{(m)} \geq P_T / G_{ij}^{(m)} \tag{4}
\]

Based on the above analysis, we can model the interference as follows.

2.1.1 PR-to-CR interference: We ignore the CR-to-CR interference because at any arbitrary time on a specific channel only one transmission is active. In transmission, each CR transmits data based on a power level which is calculated according to the current status report of PRs without causing unacceptable interference to neighbouring PRs. So, we need to take the accumulated possibility of status-flipping into consideration of all idle PRs that are closer to the target CR.

Assuming that around a CR node \( i \) there are \( L_i \) neighbouring PR nodes that can be labelled as \( l = 1, 2, \ldots, L_i \), from close to far. Denote its power mask set on channel \( m \) by \( P_{\text{mask}}^{(m)} = (p_{1}^{(m)}, p_{2}^{(m)}, \ldots, p_{L_i}^{(m)}) \), where \( p_{j}^{(m)} = P_l / G_{lj}^{(m)} \) for \( j = 1, \ldots, L_i \) and \( G_{lj}^{(m)} \) is the path loss from CR \( i \) to PR \( j \) and \( G_{lj}^{(m)} \geq \cdots \geq G_{L_i}^{(m)} \), the \( L_i \) PR neighbour is the one beyond which the CR-to-PR interference is always smaller than \( P_l \) even if the CR is transmitting on its full power \( P_{\text{max}} \). We can define \( P_{l+1}^{(m)} = P_{\text{max}} \). Define the channel-usage profile at the \( n \)th reporting time as an \( L_i \)-dimensional vector \( S^{(n)} = (s_1^{(n)}, s_2^{(n)}, \ldots, s_{L_i}^{(n)}) \), for each entry \( s_j^{(n)} \), if PR \( j \) is receiving the \( n \)th channel usage report at time \( \tau_n \), then \( s_j^{(n)} = 1 \), else \( s_j^{(n)} = 0 \). In which \( \tau_n \) denotes the time that the \( n \)th channel usage report is sent out, so at time \( \tau_n \) the status of PR \( j \) can be represented with a random process \( x_j^{(n)} \). If \( j \) is transmitting then \( x_j^{(n)} = 1 \); else \( x_j^{(n)} = 0 \). From \( \tau_n \) to next reporting time \( \tau_{n+1} \), the status of \( j \) can be described with a new random variable

\[
\xi_j^{(n)} = \begin{cases} 
1, & \text{if } x_j^{(n)}(t) = 1, \text{ for some } t \in [\tau_n, \tau_{n+1}] \\
0, & \text{if } x_j^{(n)}(t) = 0, \text{ for all } t \in [\tau_n, \tau_{n+1}] 
\end{cases} \tag{5}
\]

It is possible that the status of PR \( j \) changes between \( \tau_n \) and \( \tau_{n+1} \). Actually, there are four possible cases: (1) \( \xi_j \) keeps
0. The probability of case 1 can be calculated as follows

\[ Pr\{\xi_i^m = 0 | x_i^m = 0\} = Pr\{T_{OFF} > T \} \]
\[ = 1 - \int_0^T f_{OFF}(t) \, dt \] (6)

(2) \( \xi_j \) changes from 0 to 1 after \( T_{OFF} \). The probability of this case is

\[ Pr\{\xi_i^m = 1 | x_i^m = 0\} = Pr\{T_{OFF} \leq T \} \]
\[ = 1 - \int_0^T f_{OFF}(t) \, dt \] (7)

(3) \( \xi_j \) changes from 1 to 0 after \( T_{ON} \). The probability is

\[ Pr\{\xi_i^m = 0 | x_i^m = 1\} = Pr\{T_{OFF} > T \} = 0 \] (8)

(4) \( \xi_j \) keeps 1 between \( \tau_n \) and \( \tau_{n+1} \). The probability is

\[ Pr\{\xi_i^m = 1 | x_i^m = 1\} = Pr\{T_{OFF} > T \} = 1 \] (9)

where \( T_{OFF} \) is the forward recurrence time of the OFF status, 
\( T = \tau_{n+1} - \tau_n \) is the sensing period, \( f_{OFF}(t) \) is the probability density function (pdf) of \( T_{OFF} \) that can be derived from the pdf of the OFF period \( f_{ON}(\tau) \). Actually, according to \[3\]
\[ f_{OFF}(t) = \text{calc}(f_{OFF}(t) = 1 - \int_0^{\tau} f_{ON}(\tau) \, d\tau/ \int_0^{\tau} f_{ON}(\tau) \, d\tau). \]

For a channel \( m \) with two statuses, the probability of the status switch can be modelled with a Gilbert–Elliott model. In this model, the probability of the channel being in ON and OFF is \( \pi_{ON} \) and \( \pi_{OFF} \), respectively

\[ \pi_{ON} = \frac{\mu_{NF}}{\mu_{NF} + \mu_{FN}}, \quad \pi_{OFF} = \frac{\mu_{NN}}{\mu_{FF} + \mu_{NN}} \] (10)

in which \( \mu_{NF} \) is the probability when \( \xi_i \) changes from 1 to 0 in \( [\tau_n, \tau_{n+1}] \), \( \mu_{FN} \) is the probability when \( \xi_i \) changes from 0 to 1, \( \mu_{NN} \) is the probability when \( \xi_i \) remains 1 and \( \mu_{FF} \) is the probability when \( \xi_i \) remains 0. It can be easily obtained according to (6)–(9). So, all the interference power for \( L_i \) PR nodes over channel \( m \) is

\[ P_{PR-CR}^{(m)} = \sum_{l=1}^{L_i} P_{l}^{(m)} G_{l}^{(m)} \pi_{ON}^{(m)}, \quad m = 1, \ldots, M \] (11)

2.1.2 PR-to-PR interference: According to the interference of PR-to-CR, we can obtain the interference at CR nodes caused by PR nodes and a probable channel set can be available. However, a PR node may suffer interference from other PRs. Similar to PR-to-CR interference, the PR-to-PR interference at a PR node \( k \) over channel \( m \) is the aggregate interference of its neighbours on channel \( m \)

\[ P_{PR-PR}^{(m)} = \sum_{l=1, l \neq k}^{L_i} P_{l}^{(m)} G_{l}^{(m)} \pi_{ON}^{(m)}, \quad m = 1, \ldots, M \] (12)

where \( L_k \) is the number of \( k \)'s PR neighbours.

2.1.3 Guaranteeing outage probability for PR: The intra-channel state information of the CR is perfectly known; however, because of loose cooperation between the CR and PR nodes, the inter-channel information is only partially available to a PR transmitter. In this paper, we apply average interference power tolerance that can tolerate the interference over channel \( m \) within a certain threshold and each PR node works under an outage probability \( 1 - \beta \) \( (\beta \leq 1) \) [3]. It guarantees that the transmission of PR will not be disturbed with the probability by \( \beta \). Therefore the maximum allowable transmission power \( P_{\beta}^{(m)} \) that CR node \( i \) can transmit over channel \( m \) such that all communicating PR users within the communication range of the transmitting CR are not affected by probability \( \beta \)

\[ Pr\{P_{PR-CR}^{(m)} + G_{i}^{(m)} P_{i}^{(m)} \leq F_{L}^{m}\} = \beta \] (13)

Accordingly, the maximum allowable transmission power for CR \( i \) over different channels can be obtained as a vector \( P_{\beta}^{(m)} = \{P_{1}^{(1)}, P_{2}^{(2)}, \ldots, P_{M}^{(M)}\} \).

2.1.4 Rate requirements: Let \( r_i \) denote the transmission rate supported by CH \( i \) \((i = 1, 2, \ldots, M) \). When a CR node \( i \) has data to be transmitted to another CR node \( j \) at a total rate \( R_i \), the supported aggregate rate of all idle channels should be less than \( R_i \). In other words, \( R_i \leq \sum_{m=1}^{M} P_{\beta}^{(m)} \) in which \( M_i \) is the number of idle channels can be assigned to node \( i \)'s transmission. If the required rate is much smaller than the aggregate rate of idle channels, the number of the channels allocated to CR \( i \) should be as small as possible.

2.2 Polychromatic set theory

PST is a novel system theory proposed by Prof Pavlov in 2002 [23–26]. The key idea of PST is to use a standardised mathematical model to simulate different objects. Owing to the availability of the standardised mathematical model, PST has made significant progress in problem formalisation. The method has a significant advantage, which has also been considered as a contribution to theoretical development in systems theory. From 2002, the PST has been developed and the whole framework of this method has been modified. The theory, techniques and approaches of polychromatic set can play an important role in network simulation, problem optimisation, process modelling and process optimisation [27].

2.2.1 Basic concepts in polychromatic set theory: In PST, a set \( A = \{a_1, \ldots, a_n\} \) and its element \( a_i \) can be pigmented with different colours to represent the research objects as well as the properties of its elements, in which different colours are constituents of a colour set.

Definition 1: Individual colour set \( F(a_i) \) of the element \( a_i \) denotes the colour set which belongs to the element \( a_i \). The individual colour of all elements in set \( A \) can be represented by a Boolean matrix as \([c_{ij}]_{n \times 4} F(a_i) = [A \times F(a_i)]\), in which if \( F_j \in F(a_i) \) and \( F(a) = \vee_{m=1}^{M} F(a) \), then \( c_{ij} = 1 \).

Definition 2: Unified colour set \( F(A) \) denotes the colour set of the entirety of a polychromatic set (the unified object), and \( F(A) = <F_1, \ldots, F_j, \ldots, F_m> \), in which \( F_j(j = 1, \ldots, m) \) denotes the constituties of \( F(A) \). Actually, all the individual colours construct the unified colour, that is to say that the existence of unified colour sets \( F(A) \) is based on the existence of the individual colour of element \( a_i \). A unified
colour set can be depicted as Boolean matrix \( \langle a_{ij}, F(\ldots, A) \rangle = [A \times F(A)] \).

**Definition 3**: Entity. When element \( a_i \in A \) and if \( F_i(A) \) is also available, then the entity \( A_j(F_i) = (a_{i1}, a_{i2}, \ldots, a_{in}) \). The constituents of elements \( a_i \in A \) are called the entity of the unified colour. The constituents of the elements of the entity which guarantees the existence of all unified colours of PS can be represented by the following Boolean matrix as \( \langle a_{ij}, A \times F(_{\ldots, i}) \rangle = [A \times A(F)] \), in which \( A(F) \) is the composition of all the entities of unified colour.

The Boolean matrix that represents the relationship between elements and colour sets can be named as polychromatic sets matrix (PSM).

**2.2.2 Operations in polychromatic sets**: There are two basic operations in polychromatic set theory: logical operation and numerical operation. Actually, the colour of an object is a Boolean value [25]. So, the logical operations in PS include conjunction operation and disjunction operation [25]. In PS the individual colour can represent the quantity as well as logical value. Here the numerical of colour \( F_i \) can be represented as \( |F_i| \), for example \( |F_1| = 12 \), and the logical value of the colour \( F_i \) is \( F_i = 1 \). The core idea of PST is to process the relationships between individual colour and unified colour, according to the existences conditions, conjunction PS and adjunction PS [25].

**3 PS-based spectrum access**

The basic idea to calculate the power mask is to adapt its interference range to the activity of neighbouring PRs. The interference caused by PRs and CRs can be easily calculated according to the model in Section 2 when the channel parameters are sensed. According to the characteristics of the channel, such as guarantee outage, we can easily calculate the form of the activity of CR nodes. The power mask is the model proposed in Section 2 when the channel parameters are sensed. Both the power mask and the model can be easily forwarded to neighbouring CR nodes. Actually, PSMs are also very flexible and suitable for calculating the optimal channels. In this section, we proposed two algorithms: Algorithm 1 (Fig. 1), available channel alignment algorithm is enable to find all available channels for a CR node, and Algorithm 2 (Fig. 2), channel assignment algorithm, can select the optimal channels that can maximise the channel utilisation of a CRN.

We will illustrate the main idea of PSM-based distributed dynamic spectrum access method by an application example. In a CRN, the CR nodes can share spectrum with PRN. In our PS-based model, we embed all the collected information into a PS matrix, with which CR node can calculate the available channels at a CR node with the proposed algorithm below. After exchanging PSMs with other CR nodes, each CR can embed the CRN information into its PSMs. Finally, each CR node can make its a reasonable decision locally. Owing to the high computing ability of PSMs, the available channels and routes can be obtained rapidly. On the other hand, the communication overhead in this model would be smaller since the results of each CR node have been embedded into a PSM. The optimal solution for spectrum access may be achievable by all CR nodes.

**3.1 Spectrum sensing**

A CR node periodically senses the spectrum for every access period \( T_p \) [28]. If the channel is indicated idle it will be marked OFF, and the CR node will access the spectrum, as shown in Fig. 3. However, during the transmission, the CR node will not be aware of the PR node until the next spectrum sensing is performed. Actually, the spectrum cannot be sensed and accessed simultaneously by the same CR; therefore, if a PR starts transmission between two sensing points, it will be interfered by the transmission of

**Fig. 1 Algorithm 1: available channel alignment algorithm**

**Input**: All available channels at node \( s_1, s_2 \); \( \text{PS}_{CR}; \text{M}_s \)

**Output**: Selected Channels at node \( s_1 \)

**Initialization**: Calculate \( \text{PS}_{PR} \) according to Eq.(12);

for \( m = 1 \ldots M \) do

\[ P_{S^m, PS_{PR}^m} = 0 \]

end

for \( m = 1 \ldots M \) do

\[ P_{S^m, PS_{PR}^m} = 0 \]

end

Calculate the entity of \( S = M \times P_{CR}; S_{CR} \)

**Fig. 2 Algorithm 2: Channel assignment algorithm**

**3.1 Spectrum sensing**

A CR node periodically senses the spectrum for every access period \( T_p \) [28]. If the channel is indicated idle it will be marked OFF, and the CR node will access the spectrum, as shown in Fig. 3. However, during the transmission, the CR node will not be aware of the PR node until the next spectrum sensing is performed. Actually, the spectrum cannot be sensed and accessed simultaneously by the same CR; therefore, if a PR starts transmission between two sensing points, it will be interfered by the transmission of

**Fig. 3 Channel sensing period**

a An example of CRN
b Individual colour \( M \) of \( \text{PS}_{CR} \) (PS_{CR}; M)
the CR. The transmission of both PR and CR nodes will depend on the sensing period \( T_p \). Therefore the trade-off between the quality of service (QoS) performance of the PR and CR has to be considered when determining the \( T_p \).

### 3.2 Channels selection algorithm based on PSM

We first build its PSM at a CR node, which contains four sub-polychromatic sets (PS): CR node, PR node, links, channels as its element sets, respectively. It can be expressed as (14)

\[
PS_N = \{PS_{CR}, PS_{PR}, PS_1, PS_n\}
\]  

(14)

For a single CR node, its \( PS_{CR} \) is null at first. It mainly maintains the information about the whole CRN. \( PS_{CR} \) uses the CR nodes set in the CRN as its element set, \( A = \{ c_1, c_2, \ldots, c_N \} \), in which \( N \) is the number of CR nodes in a CRN. At the initial phase, a CR node does not know any information about its CR neighbours, so there is just one element in \( A \), for example, for CR node \( i \), \( A = \{ c_i \} \). In the spectrum sensing phase, when \( i \) receives the broadcast message \( RREP_{PSM} \) from other CR nodes, it will add the neighbours into \( A \). The unified colour set for \( PS_{CR} \) is defined as \( F(A) = [N, M, \text{Link}, R_t, M_{\text{FRR}}, M_{\text{idle}}] \), in which the individual colour \( N \) is the set of nodes, the individual colour \( M \) is the set of channels, the individual colour ‘Links’ is the set of links in a CRN, \( R_t \) is the bit rate that can be obtained according to Section 2.1.4, and \( M_{\text{FRR}} \) (\( M_{\text{idle}} \)) is the available channels for node \( i \), which is critical for the transmission. For example, for a CRN described in Fig. 4a, the corresponding PS set for individual \( M_{\text{FRR}} \) can be built as Fig. 4b. Similarly, all individual colours set also can be built, respectively. As shown in Fig. 4b, if the grid cell is marked with ‘\( \ast \)’, it means that for the PR node, the corresponding channel is available, for example, for node \( n_1 \), channel \( m_1 \) is available, and for node \( n_2 \), channels \( m_2, m_3, m_4 \) are all available.

Similarly, for a CR node \( i \), the \( PS_{PR} \) is a PSM with the element set as \( A = \{ p_1, p_2, p_3, p_4 \} \), which denotes PR neighbours set. The unified colour set of \( A \) is \( F(A) = [N, D_t, M, \beta, P_{\beta}, \text{SINR}] \), which includes six individual colour sets for the PR nodes set. The individual colour \( N \) represents the PR nodes set, \( D_t \) represents the distance between the PR nodes to CR node \( i \), \( M \) is the channel set of the PR nodes licensed spectrum, the fourth individual colour is outage probability which makes sure the transmission of PR will not be disturbed with the probability by \( \beta \). With \( \beta \), the maximum allowable transmission power \( P_{\beta} \) can be computed for each channel according to (13). The SINR is the received signal strength, which is calculated according to the spectrum sensing message. It is easy to understand that \( A(m_1) = \{ n_1 \} \) is the entity of the unified colour set \( A(F^1) \) and it shows the nodes who are using the channel \( m_1 \). Without losing generality, the entity of the unified colour set \( A(F^m) \) can be easily obtained according to \( PS_{PR} \).

For example, as shown in Fig. 5, a CR node has four PR neighbours. Fig. 5b is one of its PS with an individual colour for channel \( M \), in which if the PR node \( p_1 \) works, on channel \( m_1 \), the corresponding cell is marked with ‘\( \ast \)’. Similarly, other PSMs for different individual colour set also can be available easily.

As shown in Fig. 6, \( PS_{l} \) is the PS matrix based on links in a CRN. Here, \( A = \{ l_1, l_2, \ldots, l_l \} \) includes all the information about the links. The unified colour set of \( PS_{l} \) is \( F(A) = [M, \text{Src}, \text{Dst}, R] \), in which the colour set \( M \) represents the acceptable channels for a links, colour set \( \text{Src} \) represents the source node of the link, colour set \( \text{Dst} \) represents the destination node of the link and colour set \( R \) denotes the maximum rate on the link.

For the set of channels, \( A = \{ m_1, m_2, \ldots, m_M \} \) (\( M \) is the number of channels used in the whole CRN). The corresponding PS matrix \( PS_{m} \) can be defined with \( (A \times F(A)) \). \( F(A) = [N, f, k, \alpha] \), in which colour set \( N \) represents the nodes in the links, colour set \( f \) represents central frequency sets, colour set \( k \) represents the number of potential PR interference within an area of \( r_c \).

The three above PS matrices are related. They are connected by the unified colour sets. For example, the element of matrix \( A \times F(N) \) in \( PS_{m} \) is the element set of \( PS_{l} \); therefore, all colour sets of \( PS_{l} \) can be accessed by \( PS_{m} \). Similarly, \( PS_{m} \) is connected to \( PS_{n} \) by \( F(N) \) in \( PS_{m} \). Therefore in PS, all sub PS matrices can be accessed each other by its entity of unified colour. With PS model, it is very easy to obtain the available channel list.

### 3.3 Channel access algorithms

If a CR node \( i \) wants to initiate a data transmission, it starts searching for proper channels. At the beginning \( i \) senses the spectrum: \( i \) randomly picks a channel \( m_1 \) and samples it for \( t_l \) time, then \( i \) decides whether channel is ON or OFF. If channel \( m_1 \) is ON, \( i \) randomly selects the next channel \( m_2 \). Similar to the above procedure, \( i \) samples and detects if channel \( m_2 \) is ON or OFF. Suppose \( i \) finds a channel \( m_3 \) is OFF, then \( i \) will mark the \( m_1 \) in the individual colour \( M_{\text{idle}} \) in \( PS_{CR} \). After all channels are detected, all idle channels are marked in the individual colour \( M_{\text{idle}} \) of \( PS_{CR} \). Then for each channel \( m \) in \( M_{\text{idle}} \), \( i \) sends a channel probing packet (CPP) over \( m \) using a predefined power. The receiver of other CR node \( j \) measures the strength of the received CPP and decides the maximum achievable data rate that can be

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**Fig. 4** An illustration CRN and its individual colour set

- **a** An example of links in CRN
- **b** Individual colour \( M \) of \( PS_{PR}(PS_{PR}, M) \)
supported by channel \( m \). All the PSMs can be built according to their relations as we discussed in Section 2. The details of this procedure are illustrated in Algorithm 1 (see Fig. 1).

Using Algorithm 1 (Fig. 1), the available channels on each CR node can be obtained. Actually, the results of algorithm only show the available channels that determine the node locally. If a link is established over a channel, it will need to negotiate by the source node and the destination node of the link. Therefore we designed Algorithm 2 (Fig. 2) to obtain the channels that fully support the links.

It is easy to see that in PSMs each entry is either zero or one, so it causes a very low communication load in a CRN. Regarding the complexity of the scheme, both Algorithms 1 and 2 use only logical operations (conjunction operation and disjunction operation). Therefore the worst-case time complexity of Algorithms 1 and 2 are \( O(M \cdot N) \), in which \( N \) is the number of CR nodes in a CRN and \( M \) is the number of channels that can be shared by CRN and PRN (See Figs. 1 and 2).

The results of the algorithm should be updated in PS matrices, which is a list including all the nodes that satisfy the rate requirement, PR-to-CR interference requirement and PR-to-PR requirement. So, it is enough for the routing scheme requirements.

Most dynamic spectrum access approaches access the spectrum based on the system’s own judgement of the local use of the spectrum. The CR nodes always suffer from hidden node problem, whereby the CR nodes are inadvertently not able to detect the primary usage of a frequency channel. Many reasons cause the occurrence of the hidden node problem. This problem makes it hard to detect the spectrum usage. In general, the sensitivity of CRN will have to outperform PRs by a large margin. In our approach, this problem can be avoided by sharing information among CRN and PRN as mentioned in Section 2. This cooperative approach can improve the probability of detection and reduce the detection time and thus increase the overall cognitive capability of CRN. The drawback is the overhead needed to exchange sensing information.

4 Performance evaluation

In this section, we provide the performance evaluation based on the proposed method. We use cognitive radio cognitive network (CRCN) Simulator developed over NS-2 (version 2.31) to simulate our method. We consider a CRN with seven PRNs \( (M = 7) \), each channel with a 1 MHz of bandwidth) in a 1000 \( \times \) 1000 m square area. Six CR nodes

![Fig. 5 Example of PS

\( PS_{PR} \)

\( a \) An example of links in CRN

\( b \) Individual colour \( M \) of \( PS_{PR} \) (\( PS_{PR,M} \))

![Fig. 6 PS matrix of links]

\( a \) Unified colour \( M \) for \( PS \) matrix of links

\( b \) Unified colour \( n \) for \( PS \) matrix of links

![Fig. 7 Channel usage in different periods]
are deployed as shown in Fig. 4a. The transmission power of PR is 500 mW, the $P_{\text{max}}$ for a CR is 1 W, the interference $P_i = 2P_{\text{ICR}}$ tolerance as 0.12 µW. PR, licenses $CH_i$, $i = 1, \ldots, 7$.

In Fig. 7, we reported the average channel usage at CR node $n_1$ in different periods. Recall the PR nodes are deployed around $n_1$, it can be easily seen that the channel $CH_3$ has the highest utilisation and channel $CH_4$ has the lowest channel utilisation. Owing to the fact that PR1 is deployed nearest to CR1, so channel $CH_1$ licensed by PR1 has the highest priority. Actually, it is very reasonable for a CR node. It prefers to use the channel licensed by the nearer PR nodes with a higher priority. In this way, the CR node can communicate with other CR nodes with lower transmission power. It not only saves the power consumption, but also decreases the interference to other nodes. On the other hand, it can improve the agility for spectrum-aware CR networks. It can be seen from Fig. 7 that proposed method can supply each channel a steady channel usage relatively fairly.

In order to evaluate the network-level performance, we designed three scenarios: S1, S2 and S3. In S1 the four PRs coexist with five CRs, in S2 four PRs coexist with 10 CRs and in S3 four PRs coexist with 15 CRs. The four PRs were fixed at four corners of a 500 m $\times$ 500 m square as shown in Fig. 8a, and the CRs were deployed randomly in the simulation area. Two constant bit rate (CBR) connections were established between PR1 and PR3, PR2 and PR4. Each CR node can model PRs spectrum usage pattern after a sensing stage. We randomly selected a node CR2 in the square and generated a 1 kbps (packet size is 256) CBR to evaluate its interference to the two connections. Fig. 8b shows the results of packets delay. Scenarios S1’, S2’, S3’ are same as S1, S2 and S3, but use a bit rate of CBR generated by CR1 as 10 kbps (packet size is 256). It slightly causes the delay increasing. Therefore for packet data communication networks, the packet delay of incumbent PRs is a more sensitive indicator of harmful interference from CRs.

5 Conclusion

CR is the key technology that will enable flexible, efficient and reliable dynamic spectrum access by adapting the radio’s operating characteristics to an underutilised licensed spectrum. In this work, we first derive transmission power on a CR node by considering both the PR-to-CR interference and PR-to-PR interference under specific outage probability. Furthermore, according to the transmission power under different outage probability, a proper channel selection method is presented by using the polychromatic set theory. Based on the proper channel list, we develop a PSM-based routing scheme that can select the highest efficient channel to build multipath from the source CR node to destination CR node.

Future work will involve implementation of the proposed method in our testbed. It involves implementing a reconfigurable cognitive PHY layer by customising the spectrum parameters such as transmission power, propagation and so on. The architecture includes a customised CR routing module, customised CR MAC module, and a CR transceiver set module. The performance of this scheme will be further tested on the testbed.

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