Brief paper

Global stabilization of rigid formations in the plane

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A B S T R A C T

This paper considers the problem of distributed control of rigid formation shapes in the plane for multi-agent systems. A constructive perturbation method is proposed and combined with the conventional gradient control law. The proposed control law stabilizes the desired rigid formation in a global sense for all initial conditions except the case when a pair of communicating agents happen to have the same initial location. It also avoids collisions between any two communicating agents during the motion. Simulation results are provided to illustrate the effectiveness of the control algorithm.

1. Introduction

Recently, coordination control of multiple agents moving in formations has received major attention within the control community due to its broad range of applications in military missions, environmental surveys, and space missions. Among the significant literature of formation-shape maintenance problems, there are two main research directions: maintaining relative positions (Lin, Francis, & Maggiore, 2005) and maintaining relative distances (Dimarogonas & Johansson, 2008; Huang, Yu, & Wang, 2011; Oh & Ahn, 2011). In the first direction, the desired formation is specified by a given set of relative position vectors. After a suitable coordinate transformation, the formation control can be transformed to a consensus problem, and the desired formation corresponds to a single equilibrium point, which can be globally stabilized by a linear feedback control law. In the second direction, the desired formation is specified by a given set of relative distances. The main idea in this direction is to construct some potential function to characterize relative distances of the agents and drive the inter-agent distances to the desired values with the help of some negative gradient algorithm. Then, the desired formation shape is reached if the formation graph is rigid (Yu, Hendrickx, Fidan, Anderson, & Blondel, 2007). One advantage of the second approach over the first one is that collisions among agents can be avoided if the inter-agent distances are kept greater than a positive number. In general, however, multiple equilibria occur under gradient control laws, and thus the stability of the desired formation cannot be globally guaranteed unless the formation graph is a tree (Dimarogonas & Johansson, 2008, 2009). A tree contains no cycles and it is not rigid. Therefore, a tree formation shape practically cannot be constantly maintained by controlling relative distances.

The stability of rigid formations under distributed gradient control laws has been extensively studied recently. Anderson, Yu, Dasgupta, and Morse (2007) and Cao, Morse, Yu, Anderson, and Dasgupta (2007) investigated the directed triangle formation control problem for the system of three-coleader agents described by first-order integrators. Compared to the former, the latter explicitly proved that the negative gradient control law could drag the system from any non-collinear initial positions to a desired triangular formation with an exponential converging rate. Unfortunately, the results are true only for the system with three agents. When the number of agents is greater than three, Krick, Broucke, and Francis (2009) showed that the set of equilibria corresponding to the target formation becomes a three-dimensional manifold under the assumption of infinitesimal rigidity, and, by using the center manifold theory, proved that the desired formation is locally asymptotically stable under the gradient control law. Summersy, Yu, Anderson, and Dasgupta (2009) investigated undesired formation-shapes of the four-agent system originated from (Krick et al., 2009) with a complete graph, and showed that under certain acuteness conditions on the desired formation shape, any possible undesired equilibrium shape is unstable. For a system with more

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than four agents, however, there is no result on the construction of control laws that globally stabilize the system to a rigid formation in the existing literature.

In this paper, we study the undirected distributed formation control problem in the plane. A constructive perturbation method is proposed and combined with the conventional gradient control law. The new control law removes all the undesired equilibria and stabilizes the desired rigid formation in a global sense for all initial conditions except the case when a pair of communicating agents happen to have the same initial location. Moreover, it is shown that under the proposed control law no collision happens between any two communicating agents during the motion.

The rest of the paper is organized as follows. The system model and problem statement is given in Section 2. Section 3 presents the construction of our control law and the proof of the global stability of the desired formation. Simulations are included in Section 4 and the results are summarized in Section 5.

2. Preliminaries and problem statement

2.1. System model

We consider the formation control for a system with \( n \) agents. Each agent is described by a mass-point model:

\[
\dot{\mathbf{r}}_i = \mathbf{v}_i,
\]

where \( \mathbf{r}_i = [x_i, y_i]^T \in \mathbb{R}^2 \) and \( \mathbf{v}_i = [v_{xi}, v_{yi}]^T \in \mathbb{R}^2 \) are the position and velocity, respectively, of agent \( i \), \( i = 1, 2, \ldots, n \).

2.2. Rigid formation

We model the information architecture as a graph \( G = (V, E) \), where \( V = \{1, 2, \ldots, n\} \) is a set of vertices and \( E \subseteq V \times V \) is a set of edges. Denote by \( N_i \) the set of neighboring vertices of vertex \( i \). Many applications require the multi-agent system preserves a desired formation. In this paper, by the formation we mean that the distance between each pair of adjacent vertices in the graph, which corresponds to a pair of communicating agents, is equal to a given constant. Let each edge \( e_j \in E \) in graph \( G \) be assigned to a scalar parameter \( d_{ij} \), representing the desired distance which agents \( i, j \) should preserve. Let \( D = \{d_{ij} : i \in V, j \in N_i\} \). Then, the framework \( (G, D) \) represents a desired formation. The formation \( (G, D) \) is said to be rigid if the set \( D \) of the distance constraints is sufficient to maintain the formation shape. In other words, it is rigid if all distances between any pair of vertices in \( G \) remain unchanged during a continuous displacement of vertices of \( G \) provided that all distance constraints prescribed by \( D \) are satisfied. Note that the rigidity of formation \((G, D)\) defined here is equivalent to the rigidity of graph \( G \) defined in references (see, e.g., Asimow & Roth, 1979).

2.3. Gradient control law

The negative gradient algorithm is the common core of most current existing methods of stabilizing distance-based formation shape (Dimarogonas & Johansson, 2008; Krick et al., 2009). Let us briefly recall the algorithm.

Let \( d_{ij} = r_i - r_j \) be the displacement from agent \( i \) to its neighboring agent \( j \). Denote by \( \|r_{ij}\| \) the Euclidean distance between agents \( i \) and \( j \) with \( j \in N_i \). Different distance-based potential functions are proposed for formation control in references. Without loss of generality of the analysis and the design method given in the rest of the paper, we define the potential function between agent \( i \) and its neighboring agent \( j \) as follows:

\[
V_i(r_{ij}) = \frac{(\|r_{ij}\|^2 - d_{ij}^2)^2}{\|r_{ij}\|^2}.
\]

The total potential function of agent \( i \) is given by

\[
V_i = \sum_{j \in N_i} V_i(r_{ij}).
\]

Obviously, \( V_i \) is well-defined only if \( \|r_{ij}\| \neq 0 \). Moreover, the potential function is equal to zero if all the neighbors are located apart from agent \( i \) by the distance required by the desired formation, and goes to infinity if any of the neighbors approaches agent \( i \) with zero-distance or departs from agent \( i \) with infinite-distance.

With the velocity of each agent as the control input, the gradient control law is given by

\[
\mathbf{v}_i = -\nabla V_i, \quad i = 1, 2, \ldots, n,
\]

where

\[
\nabla V_i(r_{ij}) = \left( \frac{\partial V_i(r_{ij})}{\partial r_{ij}} \right).
\]

The set of equilibria of the control system is given by:

\[
\mathcal{E} = \{ r : \nabla V_i = 0, i = 1, 2, \ldots, n \},
\]

where \( r = [r_1^T, r_2^T, \ldots, r_n^T]^T \in \mathbb{R}^{2n} \) is the position vector of the overall system. Obviously, \( V_i = 0 \) when \( \|r_{ij}\| = d_{ij} \), for \( i = 1, 2, \ldots, n \) and \( j \in N_i \). Therefore,

\[
\mathcal{E}_i = \{ r : \|r_{ij}\| = d_{ij} \} \subset \mathcal{E}
\]

is the set of equilibrium points representing the desired formation. The desired formation is said to be locally stable if the system trajectory asymptotically approaches \( \mathcal{E}_i \) for all initial states \( r(0) \) in a neighborhood of \( \mathcal{E}_i \); it is said to be globally stable if the system trajectory asymptotically approaches \( \mathcal{E}_i \) for all initial states in \( \mathbb{R}^{2n} \). Note that the well-definedness of \( V_i \) requires \( \|r_{ij}\| \neq 0 \). So we need to assume that the initial states of the agents do not belong to the set \( F = \{ r(0) \in \mathbb{R}^{2n} : r_i(0) = r_j(0), \exists i \in \{1, \ldots, n\}, \exists j \in N_i \} \). \( F \) is not an invariant set of the system. As a matter of fact, when the initial state happens to be in \( F \), the value of the potential function goes to infinity and thus the system trajectory will soon leave the set \( F \) under any negative gradient algorithm. So, we still say the desired formation is globally stable if the system trajectory asymptotically approaches \( \mathcal{E}_i \) for all initial states \( r(0) \in \mathbb{R}^{2n} \) except \( F \).

If the formation graph is a tree, then \( \mathcal{E}_i \) contains the unique equilibrium corresponding to the desired formation, which is globally stable (Dimarogonas & Johansson, 2008). Using the center manifold theory (Krick et al. 2009) demonstrated that if the formation is infinitesimally rigid, the desired equilibrium set is locally stable. Generally, however, when the formation graph contains cycle(s), there exists unexpected equilibrium points, and the desired formation is thus not globally stable under gradient control laws.

2.4. Control goal

The formation control problem considered in this paper is to design control law

\[
\mathbf{v}_i = h_i(r_{ij}), \quad j \in N_i,
\]

such that for all initial conditions \( r(0) \in \mathbb{R}^{2n} \) except \( F \), the multi-agent system can globally stabilized at the formation \((G, D)\), i.e.

\[
\lim_{t \to \infty} (\|r_{ij}(t)\| - d_{ij}) = 0,
\]

and no collision happens between each pair of adjacent agents, or say, there does not exist a time \( t = t_i > 0 \) so that

\[
\|r_{ij}(t_i)\| = 0,
\]

where \( i = 1, 2, \ldots, n, j \in N_i \).
3. Global stabilization of rigid formations

3.1. Main result

Firstly, let us give an intuitive analysis of the control law (4). At the steady state, we have \( \nabla V_i V_j = 0 \). If agent \( i \) has only one neighbor \( j \), \( \nabla V_i V_j = 0 \) will result in the distance between agent \( i \) and agent \( j \) converging to the desired one. However, when the agent \( i \) has more than two neighbors, say, \( s \) neighbors, the movement direction of agent \( i \) is determined by the synthesis of \( s \) vectors \( \nabla V_i V_j, j = 1, \ldots , s \). Even though \( \nabla V_i V_j = 0 \), it cannot result in that each vector is zero. In other words, the distances between \( i \) and its each neighbor may not reach the desired one. Thus the desired formation is not guaranteed.

In order to achieve the globally stable formation control, we design a novel undirected formation control strategy with perturbations in the velocities of mobile agents. To describe our control law more precisely, we let denote \( \alpha_i = \|r_i\|^2 \), and

\[
\rho_{ij} = \frac{dV_j(\alpha_i)}{d\alpha_i} = \frac{\|r_i\|^4 - \|r_j\|^4}{\|r_i\|^2}. 
\]

By \( \text{sgn}(\nabla V_i V_j) \) we mean \( \text{sgn}(\langle V_i, \nabla V_j \rangle) \text{sgn}(\langle V_j, \nabla V_i \rangle) \), where \( \langle V_i, \nabla V_j \rangle \) and \( \langle V_j, \nabla V_i \rangle \) are the \( x \)-component and \( y \)-component of \( V_i, V_j \) respectively; and \( \text{sgn}(\cdot) \) is the sign function. Then, we propose the following distributed control law for the formation stabilization:

\[
v_i = -\nabla V_i V_i - k_i \sum_{j \in N_i} \rho_{ij}^2 (a_i + \text{sgn}(\nabla V_i V_j)),
\]

where the parameters are designed according to the following principles:

(p1) \( k_i > 0 \);
(p2) \( a_i = [\cos(\omega_i t) \sin(\omega_i t)]^T \in \mathbb{R}^2 \) is a unit vector, i.e., \( \|a_i\| = 1 \);
(p3) \( \omega_i \neq \omega_j, \forall i, j \in \{1, 2, \ldots , n\} \).

We see that (9) is a modified gradient control law with state-dependent perturbations. The perturbation strength (the module of the perturbation term \( k_i \sqrt{\sum_{j \in N_i} \rho_{ij}^2 (a_i + \text{sgn}(\nabla V_i V_j))} \) is mainly adjusted by \( \sqrt{\sum_{j \in N_i} \rho_{ij}^2} \) and decays to zero if and only if the distance between agent \( i \) and its neighboring agents are equal to the desired values. The direction of the perturbation is determined by the directions of \( a_i \) and \( \text{sgn}(\nabla V_i V_j) \). In particular, the direction of \( \text{sgn}(\nabla V_i V_j) \) is determined by \( \nabla V_i V_j \). More precisely, \( \text{sgn}(\nabla V_i V_j) \) takes only eight possible directions in the plane, and always in the same quadrant of \( \nabla V_i V_j \). We take \( \omega_i \) with different values for different \( i \) so that the direction of \( a_i \) changes with time \( t \) periodically and each pair of perturbation terms will never be in the same direction constantly.

The following theorem shows that under the proposed gradient control with perturbations, the equilibrium corresponding to the desired formation is unique and globally stable.

**Theorem 1.** Assume that the system (1) is driven by the control law (9) satisfying (p1)–(p3) with potential function \( V_i \) given by (2). Then, the desired rigid formation is asymptotically stable for all initial conditions except the case when a pair of communicating agents happen to have the same initial location, the velocities of all the agents converge to zero, and no collision between any pair of communicating agents happens during the motion.

3.2. Proof of Theorem 1

The proposed stabilizer is discontinuous and time-varying, then we cannot directly use the invariance-like principle for smooth non-autonomous systems (Khalil, 1996) or the non-smooth LaSalle’s invariant theory used in Dimarogonas and Johansson (2009) to analyze the stability of the closed-loop system. In this paper, we combine the Filippov’s solution theory (Filippov, 1988), Clarke’s generalized gradient (Shevitz & Paden, 1994), and the Barbalat’s Lemma (Khalil, 1996) to demonstrate the uniqueness of the equilibrium corresponding to the desired formation and the global stability of the overall system under the proposed gradient control with adaptive perturbation.

To present the proof of Theorem 1, we need to do some preparation. Let all the edges in graph \( G \) be numbered as \( e_1, e_2, \ldots , e_m \), where \( m = |E| \). Corresponding \( e_k, k = 1, 2, \ldots , m \), we have a displacement between two vertices (agents) \( r_k = r_i - r_j \), where \( i \) and \( j \) are the index numbers of the vertices of \( e_k \). Then, we can define a vector of displacements between each pair of neighboring agents as \( \tilde{r} = [\tilde{r}_1, \tilde{r}_2, \ldots , \tilde{r}_m]^T \). Denote by \( d_0 \) the distance of the displacement \( r_k \) required by the desired formation.

Let the Lyapunov function candidate be chosen as the following nonnegative function:

\[
V(\tilde{r}) = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \sum_{j \in N_i} V_i(r_i).
\]

Based on the fact that \( V_i(r_i) = V_j(r_j) = -V_j V_j \), we have

\[
\frac{d}{dt} \sum_{i=1}^{n} V_i = 2 \sum_{i=1}^{n} V_i V_i V_i.
\]

Note that \( V(\tilde{r}) \) is smooth and hence regular, its generalized gradient (Clarke, 1983) is a singleton which is equal to its usual gradient everywhere in the state space: \( \partial V = \{V V \} \) (Shevitz & Paden, 1994). Along with (9), (10)–(11) we have

\[
\dot{V}(\tilde{r}) = 2 \sum_{i=1}^{n} \langle V_i, V_i \rangle \tilde{r}_i \subset -2 \sum_{i=1}^{n} \|V_i V_i\|^2 - 2 \sum_{i=1}^{n} \sum_{j \in N_i} \rho_{ij}^2 (a_i + K[\text{sgn}(\nabla V_i V_j))].
\]

where \( K[f](x) \) is called Filippov set-valued mapping defined in detail in Filippov (1988). In obtaining (12) we have used Theorem 1 of Paden and Sastry (1987) to calculate the inclusions of the Filippov set. Since \( \nabla V_i V_j K[\text{sgn}(\nabla V_i V_j) = \langle |(\nabla V_i V_j) x_i | + |(\nabla V_j V_j) y_j \rangle \) (Paden & Sastry, 1987), we have

\[
\dot{V}(\tilde{r}) = -2 \sum_{i=1}^{n} \|V_i V_i\|^2 - 2 \sum_{i=1}^{n} \tilde{r}_i \sum_{j \in N_i} \rho_{ij}^2 (a_i + K[\text{sgn}(\nabla V_i V_j))]
\]

\[
\leq -2 \sum_{i=1}^{n} \|V_i V_i\|^2 \leq 0.
\]

Therefore, we have the following lemma.

**Lemma 1.** Given any positive number \( W_0 < \infty \), the set

\[
S = \{ \tilde{r} : V(\tilde{r}) \leq W_0 \}
\]

is compact and positively invariant for the trajectories of the closed-loop system (1) driven by the controller (9).
Proof. From the definition of $V$ we know that $\|\hat{r}_i\|$ are bounded if $V(\hat{r}) \leq W_0 < \infty$. Thus, the set $S$ is compact. Moreover, since $V(\hat{r}) \leq 0$, we have $V(\hat{r}(t)) \leq V(\hat{r}(0))$, where $V(\hat{r}(0)) \leq W_0$. According to the definition of positively invariant set (Khalil, 1996), we can conclude that $S$ is positively invariant for the trajectories of the closed-loop system. □

Now, we use Barbacal’s Lemma to demonstrate that from any initial state in $S$, the system trajectory converges to the invariant set $\Omega = \{\hat{r} : \nabla_r V_i = 0\}$.

Lemma 2. Consider system (1) driven by the controller (9) with potential function given by (2), and starting from any initial state in $S$ given by (14). Then all the system trajectories converge to the invariant set $\Omega = \{\hat{r} : \nabla_r V_i = 0\}$.

Proof. Since $V(\hat{r}(t))$ is non-increasing and bounded from below by zero, we conclude that it has a limit $V(\hat{r}(\infty))$ as $t \to \infty$. Let $W(\hat{r}(t)) = 2 \sum_{i=1}^{\infty} \|\nabla_r V_i\|^2$, integrating the Eq. (13), we have

$$\lim_{t \to \infty} W(\hat{r}(t)) = 0,$$

which means $\lim_{t \to \infty} \nabla_r V_i = 0$. Along with Lemma 1, we conclude that all the system trajectories converge to the invariant set $\Omega = \{\hat{r} : \nabla_r V_i = 0\}$. □

Now, we can complete the proof of Theorem 1.

Proof of Theorem 1. Firstly, we prove that with the choice of the potential function given in (2), any two communicating agents do not collide with each other and there is a minimum separation distance between them if the system starts from any initial states within $S$.

For any $\hat{r}(0) \in S$, the time derivative of $V(\hat{r})$ remains non-positive for all $t \geq 0$, by virtue of (13). Hence, $V(\hat{r}(t)) \leq V(\hat{r}(0)) \leq W_0 < \infty$ for all $t \geq 0$.

Moreover, since $V(\hat{r}) = \sum_{i=1}^{n} \sum_{j \in \Omega} V_j(r_j)$, we have $V_j(r_j) \leq W_0$, so that

$$\frac{-\sqrt{W_0} + \sqrt{W_0 + 4d^2}}{2} \leq \|r_j(t)\|$.

It is obvious that $\frac{-\sqrt{W_0} + \sqrt{W_0 + 4d^2}}{2} > 0$. Therefore, no collision happens between any two adjacent agents.

Next, we show that in the invariant set $\Omega$, all the agents’ velocities are equal to zero, and the desired equilibrium is the unique and stable one of the overall systems.

By Lemma 2 we have

$$\nabla_r V_i = \sum_{j \in \Omega} 2 \rho_j \hat{r}_j \to 0$$

as $t \to \infty$, and hence,

$$v_i = -k_i \sum_{j \in \Omega} \rho_j, \quad \forall i \in \Omega$$

for $i = 1, 2, \ldots, n$, which contains no sign terms. Since $v_i$ contains no sign terms in the invariant set, and $\rho_i$ and $r_j$ are differentiable functions, from (16) it should follow that

$$d(\nabla_r V_i) = \sum_{j \in \Omega} (v_i - v_j)\rho_j,$$

which implies

$$\frac{dV_i}{dt} = 2 \sum_{j \in \Omega} (v_i - v_j)\rho_j,$$

It is obvious that the vectors $dV_1$ and $dV_i$ cannot be collinear constantly (the direction of $dV_1$ is determined by $r_1(t)$ and the direction of $dV_i$ is determined by $v_i(t) - v_j(t) = \hat{r}_i(t)$). Therefore, from (18) it follows that $dV_1 \to 0$, i.e.,

$$v_i(\rho_j + \ldots + \rho_j) - v_j \rho_i \to 0$$

for $i = 1, 2, \ldots, n$, where $j_1, \ldots, j_m$ are the neighbors of agent $i$. Since each $a_i$ has a distinguished rotating frequency $a_i$, any pair of vectors $v_i, v_j, \ldots, v_j$, given by (17) cannot have the same direction constantly. And hence, (19) holds only if $\rho_j \to 0$ or $v_j \to 0$ for all $j \in \Omega$. Observing (17) we know that $v_j \to 0$ also implies $\rho_j \to 0$. So, we have

$$\rho_j \to 0, \quad \forall i \in \Omega, \forall j \in \Omega.$$

By (17), this implies that all the agents’ velocities tend to zero, and all the edges in graph $G$ converge to the desired distances. □

4. Simulations

In this section we provide some simulation examples to support our theoretic result on the global stability of the proposed control law.

Let us consider a system of 6 agents described by (1). The information structure of the system is given by a (redundantly) rigid graph $G$ shown in Fig. 1.

Case 1: Let the desired distance between each pair of adjacent agents be $d_{12} = d_{23} = d_{34} = d_{45} = d_{56} = d_{61} = 3$. $d_{13} = d_{46} = d_{35} = 3/\sqrt{3}$, $d_{25} = 6$. This set of distances forms an
equilateral-hexagon formation. We set the agents' initial positions randomly in region of $x \in [0, 10], y \in [0, 10]$. Firstly, we use the conventional gradient control method (let $k_i = 0$ for $i = 1, \ldots, 6$). More than 100 simulation experiments show that there are about 34% tests converge to undesired formations. But when applying our controller (9) with $k_i = 20$ for $i = 1, \ldots, 6$ and $\omega_1 = 0.02, \omega_2 = 0.62, \omega_3 = 0.82, \omega_4 = 1.02, \omega_5 = 1.41, \omega_6 = 2.05$, there is no false case for random initial states! The movement trajectories of the agents and the relative distances $\|r_{ij}\|$ of such an experiment are shown in Figs. 2 and 3, respectively, which demonstrate that six agents achieve the desired formation. Fig. 4 demonstrates that the velocities $v_{xi}$ along the $x$-axis and $v_{yi}$ along the $y$-axis all tend to zero at the steady state.

Now, we set the agents' initial positions as:

$$r_1(0) = [1, 1]^T, \quad r_2(0) = [2, 2]^T, \quad r_3(0) = [3, 3]^T, \quad r_4(0) = [4, 4]^T, \quad r_5(0) = [5, 5]^T, \quad r_6(0) = [6, 6]^T.$$ 

Note that such initial positions are collinear. Under the standard gradient control law (see, e.g., Krick et al., 2009), they remain collinear forever, and the system cannot converge to the desired formation. Now, we use the control law given by (9). The movement trajectories of the agents are shown in Fig. 5, which demonstrates that six agents achieve the desired formation. Fig. 6 demonstrates that the velocities $v_{xi}$ along the $x$-axis and $v_{yi}$ along the $y$-axis all tend to zero at the steady state.

Case 2: Let the desired distances between each pair of adjacent agents be $d_{12} = d_{23} = d_{34} = d_{45} = d_{56} = 3, d_{61} = 15, d_{13} = d_{45} = d_{64} = 6, d_{25} = 9$. It is easy to check that the desired formation is on a line. Line formations are required in some special missions, for example, sometimes small satellites should form a line formation for earth observation. Such a line formation with distance constraints cannot be stabilized even locally by the
standard gradient control law given in the existing references, e.g., Cao et al. (2007), and Krick et al. (2009). Now, we apply the controller proposed in this paper. The control parameters $\omega_i$ and $k_i$ are the same as given in Case 1. The agents’ initial positions are given as:

\[
\begin{align*}
r_1(0) &= [1, 1]T, \\
r_2(0) &= [2, 0]T, \\
r_3(0) &= [10, 13]T, \\
r_4(0) &= [1, 2]T, \\
r_5(0) &= [2, 0.2]T, \\
r_6(0) &= [3, 0.3]T.
\end{align*}
\]

From the movement trajectories of agents shown in Fig. 7 we can see that six agents achieve the desired line formation and never collide between each other. Fig. 8 demonstrates that the velocities $v_{xi}$ along the $x$-axis and $v_{yi}$ along the $y$-axis all converge to zero at the steady state.

5. Conclusions

This paper has solved the problem of distributed and global stabilization of rigid formations in the plane for a group of mobile agents. With the help of the non-smooth analysis theory we prove that any desired rigid formation can be globally stabilized by the proposed control law, and the velocities of all agents converge to zero without collision of adjacent agents during the motion. Currently, a bi-directional graph is assumed for the information structure of the agents. How to construct a globally stabilizer for directed graph formation is still a challenging problem for future study.

References


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