Analysis of the Frequency Offset Effect on Random Access Signals

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Abstract—Zadoff-Chu (ZC) sequences have been used as random access sequences in modern wireless communication systems, replacing the conventional pseudo-random-noise (PN) sequences due to their superior autocorrelation properties. An analytical framework quantifying the ZC sequence’s performance and its fundamental limitation as a random access sequence in the presence of frequency offset between the transmitter and the receiver is introduced. We show that a ZC sequence’s perfect autocorrelation properties can be severely impaired by the frequency offset thereby limiting the overall performance of the random access signals formed from these sequences. First, we derive the autocorrelation function of these random access sequences as a function of the frequency offset. Next, we introduce the concept of critical frequency offsets and the spectrum associated with a ZC sequence set to characterize the frequency offset properties of the random access signals. Finally, we demonstrate that the frequency offset immunity of a ZC sequence set can be controlled by shaping the spectrum of the ZC sequence set.

Index Terms—Random access signal, Zadoff-Chu sequence, frequency offset, critical frequency offset, Zadoff-Chu sequence set spectrum.

I. INTRODUCTION

In wireless cellular communication systems, random access signals are typically used by a device in the cellular network for initial network access or short message transmission. The design of random access signals therefore must provide multiple virtual orthogonal “channels” allowing the realization of statistical multiplexing of the access signals from different devices on a given resource. Cellular systems almost unexceptionally employ direct-sequence spreading to fulfill such a goal. In 2G and 3G systems (e.g., IS-95 [1], CDMA2000 [2], and WCDMA [3]), the pseudo-random noise (PN) sequence [4]-[6] or its variants [3], [7]-[10] are used as the spread sequences. In 4G systems (e.g., LTE [11]), the Zadoff-Chu (ZC) sequence is chosen as the spreading sequence for its improved orthogonality. These sequences are the basis for the random access channels (RACHs) that are used for initial access.

Random access signals are one of the special signals used in wireless communications, and their design has attracted a lot of research interest [12]-[15]. A random access signal typically carries access information (e.g., control and timing information) from the access device for the purpose of setting up an initial connection to an access point before the establishment of two-way data communications. For example, in the cellular network, an access device sends a random access signal to register with the network after power-up, to perform a location update after moving from one location area to another, or to initiate a call by setting up a connection from the access device to the network access point. The random access signal is also sent when the access device performs handoff from one access point to another and when the access device loses the uplink timing synchronization with the serving access point [16]. Due to their multiple functions, random access signals must possess good cross-correlation and autocorrelation properties. The first property is required for low inter-signal interference (e.g., low correlation between different random access sequences), while the second property is necessary for timing estimation [11].

The design of random access signals that minimizes the cross-correlation and therefore the inter-signal interference has been extensively studied [4]-[10]. Many types of sequences have been considered as candidate sequences. Walsh sequences, for example, have ideal orthogonality between them [3], [17]. However, perfect synchronization to the Walsh chip level among all codes is required to maintain the orthogonality. For example, a Walsh sequence of length 16 consists of 16 chips, each with a value either “0” or “1”. The chips between the Walsh sequences must be aligned in order to maintain orthogonality between them. Unfortunately, such strict chip-level synchronization is difficult to achieve in a wireless random access environment due to multi-path and variable propagation delays from access devices to the access point.

PN sequences on the other hand are not orthogonal sequences. A complex PN sequence has a periodic autocorrelation of [3], [4], [8]

$$
\gamma_{PN}(\kappa_1, \kappa_2) = \left\{ \begin{array}{ll}
\frac{1}{N} \sum_{n=0}^{N-1} x[n + \kappa_1] x^*[n + \kappa_2] = -\frac{1}{N}, & \kappa_1 \neq \kappa_2 \\
1, & \kappa_1 = \kappa_2 
\end{array} \right. \tag{1}
$$

where $N$ is the period of the PN sequence. Therefore, the cyclically shifted PN sequences, i.e., sequences with different shift offsets from the same PN sequence, have cross-correlation of $-1/N$. Clearly, these sequences do not require synchronization to maintain good cross-correlation as long as the propagation delay difference is less than the shift offset between them. Note that when the actual length of the PN sequence is not equal to the natural period of the PN sequence, the correlation between these truncated sequences is no longer...
the same as (1) but good correlation can still be retained [3] which provides desirable flexibility in sequence length selection. PN sequences are thus commonly used as random access signals, e.g., the RACH signals on the uplink (from access device to access point) in 2G cellular systems (IS-95) [1], [18] and 3G cellular systems (CDMA2000, WCDMA) [2], [3], [19], [20].

Another class of sequences, the ZC sequence, is a class of polyphase sequences defined as [21]  
\[
x_\mu(n) = e^{-j \frac{2\pi \mu}{N} n (n+1) / 2}, \quad n = 0, 1, \ldots, N - 1 \tag{2}
\]
where \( l \in \mathbb{Z} \), \( N \) (odd) is the period of the sequence. The root of the sequence (or the sequence index) \( \mu \in \{1, 2, \ldots, N-1\} \) is relatively prime to \( N \). A commonly used variant form of the ZC sequence is [22]  
\[
x_\mu(n) = e^{-j \frac{\pi \mu n (n+1)}{N}}, \quad n = 0, 1, \ldots, N - 1 \tag{3}
\]
For a fixed \( \mu \), the ZC sequence possesses an ideal periodic autocorrelation property (i.e., the periodic autocorrelation is zero for all shifts other than zero),  
\[
\gamma_{\mu\mu}(\kappa_1, \kappa_2) = \frac{1}{N} \sum_{n=0}^{N-1} x_\mu(n + \kappa_1) \cdot x_\mu^*(n + \kappa_2) = \delta_{\kappa_1 - \kappa_2},
\]
where \( \kappa_1, \kappa_2 = 0, \ldots, N - 1 \). In (4) and in what follows, modulo-\( N \) indexing is assumed. The sequences obtained from cyclically shifting the same ZC sequence are thus orthogonal, i.e., zero cross-correlation between them with each having perfect autocorrelation. However, ZC sequences are no longer orthogonal for different root values. If the length \( N \) of the sequence is a prime number, there are \( N - 1 \) different sequences with periodic cross-correlation of [23]  
\[
\gamma_{\mu\mu}(\kappa_1, \kappa_2) = \frac{1}{N} \sum_{n=0}^{N-1} x_\mu(n + \kappa_1) x_\mu^*(n + \kappa_2) = \frac{1}{\sqrt{N}}, \quad \kappa_1, \kappa_2 = 0, 1, \ldots, N - 1,
\]
where \( \mu \neq \nu, \mu, \nu = 1, \ldots, N - 1 \). Orthogonal sequences obtained by cyclically shifting a single root sequence are therefore preferred over sequences with different roots in applications where orthogonality is required. In LTE, a set of random access sequences are obtained by cyclically shifting the original ZC sequence [22], [24], similar to the way that a set of PN sequences are created as previously described,  
\[
\{x_{\mu, \kappa}, \kappa \in K\}
\]
where \( K \) contains the set of cyclic shifts relative to the original ZC sequence (3) of root \( \mu \) for each of the created sequences.

Non-orthogonal sequences generated from different root sequences are used only when the number of orthogonal sequences from the same root is insufficient. ZC sequences with the same root are therefore the main focus of this paper. Unless the circumstances indicate otherwise, ZC sequences with the same root are implied in the sequel. The root parameter \( \mu \) can then be dropped for the purpose of notational simplification.

Although ZC sequences have been studied extensively, almost all of the previous studies of ZC sequences assume there is no frequency offset between the transmitter and the receiver. The ideal periodic autocorrelation property of the ZC sequences is derived under this assumption [21], [25]-[28]. There is little literature that addresses the effect of frequency offset on the autocorrelation of the ZC sequences [29]. Unfortunately, frequency offset does have a profound impact on the autocorrelation characteristics of a ZC sequence and hence on the orthogonality/cross-correlation property of the sequences created from a ZC sequence with the same root as well. Consequently, the properties of ZC sequences as random access sequences are affected also. Moreover, in practical wireless communication scenarios, the frequency offset between transceivers is inevitable due to the accumulated frequency uncertainties at the access device transmitter and the access point receiver as well as the Doppler spread resulting from the mobility of the access device [29], [30]. In fact, the effects of frequency offset on the performance of the ZC sequences were not identified in the early stages of LTE standardization and remedies were later added to improve the robustness of the RACH signal against frequency offset, by adding constraints in the sequence generation procedure and dividing sequences into high speed/Doppler and low speed/Doppler sets [22]. The high speed sets are used for deployments where high mobility access devices are expected and the low speed sets are designed for stationary environments. The construction of these sets was mostly based on the explicit numerical evaluations and simulations of the correlation between sequences.

In [29], the superiority of ZC sequences with the same root over PN sequences when used as random access sequences is clearly explained. The effect of frequency offset on the timing and detection performance of ZC sequences is identified and evaluated. The aim of this paper is to provide an analytical point of view of the effect of the frequency offset on the characteristics of the ZC sequences as random access sequences. In particular, we derive a closed-form expression of the correlation between two cyclic-shifted ZC sequences with frequency offset. We then use it to study the inter-sequence interference properties and to evaluate the average and upper bound of the inter-sequence interference of a given sequence set. Furthermore, we introduce the concept of the critical frequency offset associated with a ZC sequence pair and the spectrum of a ZC sequence set that we show to completely characterize the frequency offset property of the sequence set. Throughout this paper, we use LTE RACH as a framework for ease of description and discussion.

The remainder of this paper is organized as follows. Section II describes the mathematical model and use of the ZC sequence as a random access signal. Section III derives the correlation of two cyclically shifted ZC sequences in the presence of frequency offset between the two sequences. Section IV analyzes the interference properties, specifically, the average and maximum interference, of a given set of ZC sequences. As a practical example, in Section V, we apply the analytical results to evaluate the characteristics of the ZC
sequence sets specified by LTE, including the sets designed for high frequency offset scenarios and the sets for low frequency offset scenarios. Finally, the paper concludes in Section VI with a review of the main results.

II. ZC SEQUENCES AS RANDOM ACCESS SIGNALS

As a random access signal, a ZC sequence is modulated onto either a CDMA waveform or an OFDM/SC-FDM waveform for transmission. In LTE, the time-continuous wideband random access signal in the SC-FDM waveform can be mathematically represented as [22]

\[ s_\kappa(t) = g_\kappa \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} x_\kappa[q] \cdot e^{-j \frac{2\pi}{N} pq} \cdot e^{j 2\pi (p+q) \Delta f_s (t-T_{CP})}, \quad 0 \leq t \leq NT_s + T_{CP}, \]

where \( x_\kappa = \{x[n+k], n = 0, 1, \cdots, N-1\} \) is the ZC sequence of root \( \mu \) with a cyclic shift \( \kappa \in K \) relative to the original ZC sequence of the same root, i.e., \( x = \{x[n], n = 0, 1, \cdots, N-1\} \), \( \Delta f_s \) is the subcarrier interval of the random access signal, \( T_s = \frac{NT_s}{N\Delta f_s} \) is the time sample interval, \( T_{CP} \) is the length of the cyclic prefix whose length is selected to be sufficiently long to accommodate the largest propagation delay of the cell, \( g_\kappa \) is a transmission gain factor, and \( \varsigma \cdot \Delta f_s \) is the starting location of the random access signal in frequency domain in the system bandwidth. The random access signal structure is depicted in Fig. 1, where the guard time is used to prevent the random access signal from protruding into the following OFDM symbol timeline as a result of the propagation delay.

The random access signals received by an access point of the network can be represented by

\[ s'(t) = \sum_{\kappa \in K'} h_\kappa s_\kappa(t - \Delta t_\kappa) e^{j 2\pi \Delta f_s t + \omega(t)}, \]

where \( \Delta f_\kappa \) is the frequency offset of access device \( \kappa \) (that uses access sequence \( x_\kappa \)) relative to the receiving access point, \( h_\kappa \sim CN(0, \sigma_\kappa^2) \) is the channel gain between the device and the access point and is assumed to be fixed in the transmission duration of the random access signal, \( \Delta t_\kappa \) is the propagation time from the access device \( \kappa \) to the access point, \( \omega(t) \) is the white Gaussian noise with power spectral density \( \sigma_w^2 \), and \( K' \subseteq K \) is the set of the access sequences that are active during the access period. To extract the access sequence, the received signal is first heterodyned to the RACH signal band, located at \( \varsigma \cdot \Delta f_s \), and can be represented as (9), shown at the bottom of the page, where \( w(t) \triangleq \omega(t) e^{-j 2\pi \varsigma \Delta f_s t} \).
We absorb the transmission gain $g_t$ and a constant phase rotation $e^{-j2\pi\Delta f_c(\Delta t_s+T_{CP})}$ into the channel gain $h_n$ to form an effective channel gain to give a simplified representation of the received signal as shown in (10) at the bottom of the previous page. The receiver then samples the signal $s(t)$ at $t = nT_s + T_{CP} = \frac{n\Delta t_s}{\tau_s} + T_{CP}$ to yield (11), shown at the bottom of the previous page. Where $\Delta \lambda_n \triangleq \Delta / \tau_s$ is the frequency offset normalized to the subcarrier interval of the OFDM symbol, and $w[n] \triangleq w(nT_s + T_{CP}) \sim CN(0, \sigma^2)$. Let $\Delta t_s = \Delta \tau_s T_s + \beta_s T_s$, where $\Delta \tau_s \in \mathbb{Z}$ and $|\beta_s| < 0.5$, we have (12), where $h_n$ includes the constant phase factor $e^{j2\pi\Delta \lambda_n \frac{n-\ell}{\tau_s}}$ in (11). We ignore $\beta_s$ (assume $\beta_s = 0$) in the following derivation for mathematical tractability. Therefore, (12) can be simplified as

$$y[n] = \sum_{\kappa \in K'} h_n x_\kappa [n - \Delta \tau_n] e^{j2\pi \Delta \lambda_n n} + w[n]$$

$$= \sum_{\kappa \in K'} h_n x_\kappa [n - \Delta \tau_n] e^{j2\pi \Delta \lambda_n n} + w[n], n = 0, 1, \cdots, N.N.$$

Assuming $\Delta \lambda_n = 0$, i.e., zero frequency offset between the transmitter and the receiver, it is clear from (13) that a timing offset between the transmitter and the receiver causes the original transmitted sequence $x_\kappa$ to become a new sequence $x_\kappa - \Delta \tau_n$. If the new sequence $x_\kappa - \Delta \tau_n$ is a valid random access sequence, i.e., $\kappa - \Delta \tau_n \in K$, the receiver (i.e., the access point) will mistake $x_\kappa$ for $x_\kappa - \Delta \tau_n$. In order to avoid the time-shifted original access sequence from becoming a new valid access sequence, the set of valid cyclic shifts $K$ must be carefully designed, i.e., the minimum spacing of the sequences in $K$ must be large enough to accommodate the largest possible propagation delay of a cell. In this paper, however, we focus on the effect of the frequency offset only, i.e., $\Delta \lambda_n, \kappa \in K$, on the performance of the ZC sequence set, $K$. We therefore ignore the timing offset by assuming $\Delta \tau_n = 0, \kappa \in K$.

As illustrated in Fig. 2, the receiver performs correlation across a pool of random access sequences, i.e., the ZC sequences with the set of shifts $K$ allocated to the access point by the network. Under the assumption of zero timing offset, and from (13), the output of correlator $x_\ell$ for each $\ell \in K$ can be represented as

$$z_\ell = \frac{1}{N} \sum_{n=0}^{N-1} y[n] \cdot x_\ell^*[n]$$

$$= \sum_{\kappa \in K'} h_\kappa \left( \frac{1}{N} \sum_{n=0}^{N-1} x_\kappa [n] e^{j2\pi \Delta \lambda_n n} \cdot x_\ell^*[n] \right) + \nu_\ell$$

Here, $\nu_\ell \triangleq \frac{1}{N} \sum_{n=0}^{N-1} w[n] \cdot x_\ell^*[n]$ is a linear combination of Gaussian variables, therefore is also Gaussian, $CN\left(0, \frac{\sigma^2}{N} \right)$, and

$$\gamma(\ell, \kappa, \Delta \lambda_n) \triangleq \frac{1}{N} \sum_{n=0}^{N-1} x_\kappa [n] e^{j2\pi \Delta \lambda_n n} \cdot x_\ell^*[n]$$

(15) is the correlation between sequences $x_\ell$ and $x_\kappa$. In the absence of frequency offset, i.e., $\Delta \lambda_n = 0$, (15) reduces to equation (4).

III. THE EFFECT OF FREQUENCY OFFSET ON THE CORRELATION OF RANDOM ACCESS SEQUENCES

In this section, we study the effect of frequency offset on the correlation property between two arbitrary cyclic-shifted ZC sequences $x_\ell$ and $x_\kappa$ given in (15). Substituting (3) into (15) yields

$$\gamma(\ell, \kappa, \Delta \lambda_n) = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} e^{-j2\pi \left(\mu(n-\ell)/(2n+m+1) + \nu \right)} e^{j2\pi \mu \Delta \lambda_n n}.$$

(16)

The squared absolute value of the correlation function $\gamma(\ell, \kappa, \Delta \lambda_n)$ is

$$\frac{1}{N^2} \sum_{\mu=0}^{N-1} \sum_{\nu=0}^{N-1} e^{-j2\pi \left(\mu(n-\ell)/(2n+m+1) + \nu \right)} e^{j2\pi \mu \Delta \lambda_n n}$$

(17)

$$= \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} e^{-j2\pi \left(\mu(n-\ell)/(2n+m+1) + \nu \right)} e^{j2\pi \mu \Delta \lambda_n n}$$

(18)

$$= \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} e^{-j2\pi \left(\mu(n-\ell)/(2n+m+1) + \nu \right)} e^{j2\pi \mu \Delta \lambda_n n}$$

(19)

$$= \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} e^{-j2\pi \left(\mu(n-\ell)/(2n+m+1) + \nu \right)} e^{j2\pi \mu \Delta \lambda_n n}.$$
where
\[ \Delta \kappa_n = \kappa - \ell + lN, \quad l = \arg \min_{l \in \mathbb{Z}} |\kappa - \ell + lN| \]

is the inter-sequence shift offset between sequences \( x_\kappa \) and \( x_\ell \). We define \( \Delta K \) to be the set of inter-sequence shift offsets associated with \( K \). Equation (17) can be further simplified as:
\[
|\gamma(\Delta \lambda_\kappa - \mu \Delta \kappa_n)|^2 = \frac{1}{N^2} \left( \frac{\sin \pi (\Delta \lambda_\kappa - \mu \Delta \kappa_n)/N}{\sin \pi (\Delta \lambda_\kappa - \mu \Delta \kappa_n)/N} \right)^2.
\]

The correlation between two sequences with a relative cyclic shift offset \( \Delta \kappa_n \) and frequency offset \( \Delta \lambda_\kappa \) is
\[
|\gamma(\Delta \lambda_\kappa - \mu \Delta \kappa_n)| = \frac{1}{N} \left( \frac{\sin \pi (\Delta \lambda_\kappa - \mu \Delta \kappa_n)/N}{\sin \pi (\Delta \lambda_\kappa - \mu \Delta \kappa_n)/N} \right).
\]

Substituting (1) into (20) yields
\[
|\gamma(\Delta \lambda_\kappa - \mu \Delta \kappa_n)| = |\gamma_{PN}| |\text{sinc}(\pi) (\Delta \lambda_\kappa - \mu \Delta \kappa_n)|.
\]

Here we use the correlation of the hypothetical or “ideal” PN sequences (i.e., PN sequences of the length \( N \) equal to its natural period and zero frequency offset) as a reference.

Utilizing the result from (14) the output at the correlator \( z_\ell \) can then be expressed as:
\[
z_\ell = \sum_{\kappa \in K} h_\kappa \gamma(\ell, \kappa, \Delta \lambda_\kappa) + v_\ell
\]
\[
= \sum_{\kappa \in K} h_\kappa \gamma(\Delta \lambda_\kappa - \mu \Delta \kappa_n) + v_\ell.
\]

We observe that:

1) The correlation between two sequences is a function of the frequency offset and the cyclic shift offset between the two sequences, rather than the absolute shift value, \( \kappa \) or \( \ell \) of the individual sequence;

2) The maximum correlation of one occurs when
\[
\Delta \lambda = \mu \Delta \kappa + l \cdot N, \quad l \in \mathbb{Z},
\]

meaning that the two shifted sequences \( x_\kappa \) and \( x_{\kappa+\Delta \kappa} \) (\( \forall \kappa \in K \)) with a shift offset \( \Delta \kappa \in \Delta K \) between them become perfectly correlated under the frequency offset given in (23). This observation can also be explained by examining the effect of frequency offset on the ZC sequence. With frequency offset \( \Delta \lambda \) the original ZC sequence \( x_\kappa \) is altered to
\[
x_\kappa (n, \Delta \lambda) = x_\kappa [n] \cdot e^{j \frac{2\pi}{N} \Delta \lambda n} = \varphi x_{\kappa+\mu p} [-1] [n] \cdot e^{j \frac{2\pi}{N} (\Delta \lambda \pm p) n},
\]

where \( \varphi = e^{j \frac{2\pi}{N} (\pm 2k p + p^2) N} \) is a constant. It is evident that when
\[
\Delta \lambda = \pm p \in \mathbb{Z}, \quad p > 0,
\]
i.e., the frequency offset equals \( \pm p \), sequence \( x_\kappa \) becomes \( x_{\kappa \mp \mu p} \). That is, a frequency offset of \( \pm p \) in effect causes a sequence to shift \( \pm \mu p \) (mod \( N \)) cyclic shifts.

It is thus clear that, in addition to the time offset, a frequency offset can also cause a cyclic shift to the received sequences. In other words, both time and frequency offsets contribute to the shift seen at the receiver. The effective cyclic shift that the receiver sees is a combination of both time and frequency offsets. This unique property of the ZC sequence results in additional timing (or frequency) uncertainties in certain applications where the ZC sequence is used to estimate the timing (or frequency) offset between the transmitter and the receiver. In contrast to the ZC sequences, a frequency offset does not cause a cyclic shift in PN sequences. In fact, the frequency offset can be directly estimated for PN based random access signal such that its effect can be minimized [31].

Equation (21) can also be rewritten as
\[
|\gamma(\Delta \lambda_\kappa - \Delta \lambda_\kappa^\dagger)| = |\text{sinc}(\pi) (\Delta \lambda_\kappa - \Delta \lambda_\kappa^\dagger)|,
\]

where
\[
\Delta \lambda_\kappa^\dagger \Delta \kappa_n = \mu \Delta \kappa_n + l \cdot N, \quad l \dagger = \arg \min_{l \in \mathbb{Z}} |\mu \Delta \kappa_n + lN|.
\]

We refer to \( \Delta \lambda_\kappa^\dagger \Delta \kappa_n \) as the critical frequency offset for a sequence pair with a relative shift offset \( \Delta \kappa \neq 0 \) at which the sequence pair have a maximum correlation of one. We define
\[
\Lambda_K^\dagger \Delta \kappa_n = \{ \Delta \lambda_\kappa^\dagger \Delta \kappa_n, \Delta \kappa \in \Delta K, \Delta \kappa \neq 0 \},
\]

which includes all the critical frequency offsets of the sequence set. We take the histogram of \( \Lambda_K^\dagger \) and refer to it as the spectrum of sequence set \( K \),
\[
S(\Delta \lambda^\dagger), \Delta \lambda^\dagger \in \Lambda_K^\dagger.
\]

\( S(\Delta \lambda^\dagger) \) thus represents the multiplicity of \( \Delta \lambda^\dagger \) in \( \Lambda_K^\dagger \). Since (29) and \( \Lambda_K^\dagger \) are equivalent, we make no distinction between them and refer to both as the critical frequency offset spectrum, or simply the spectrum, of sequence set \( K \) with inter-sequence shift offset set \( \Delta K \). It is clear that it is \( \Delta K \) and \( \mu \) that \( K \) is associated with that determines the spectrum.

Fig. 3 shows the correlation coefficients between two arbitrary sequences as a function of the frequency offset \( \Delta \lambda \) for all possible values of shift offsets \( \Delta \kappa \)'s for a ZC sequence with \( \mu = 53 \) and \( N = 839 \). If we assume the frequency offset in a typical cellular application is less than 1.25 kHz, i.e., in the range of |\( \Delta f \)| < 1.25 kHz or |\( \Delta \lambda \)| < 1 (for LTE RACH subcarrier interval of \( \Delta f_s = 1.25 \) kHz), |\( \gamma_{PN}(\Delta \lambda) \)| \( \neq 0 \) for \( |\Delta \lambda| \neq 0 \), due to the fact that the side ripples of the \( \text{sinc} \) function in (20) cause energy leakage between sequences. For every \( \Delta \kappa \in \Delta K \ (\Delta \kappa \neq 0) \), there is a corresponding critical frequency offset \( \Delta \lambda^\dagger \) at which a maximum correlation coefficient of one occurs. For example, \( |\Delta \kappa| = 95 \) corresponds to a critical frequency offset of \( |\Delta \lambda^\dagger| = 1 \) which is the least critical frequency offset among all possible critical frequency offsets. For any sequence pair with shift offset |\( \Delta \kappa \)| = 95, any small frequency offset incurs a large correlation between them. There’s also a non-zero correlation between any sequences of different shift offsets in the range of |\( \Delta \lambda \)| < 1 (|\( \Delta \lambda \)| \( \neq 0 \)), such as the sequence pairs with |\( \Delta \kappa \)| = 16 (corresponding to |\( \Delta \lambda^\dagger \)| = 9), but with much smaller order of magnitude. It is seen that a sequence pair with a smaller critical frequency offset |\( \Delta \lambda^\dagger \)| has a larger inter-sequence interference between them. However, a smaller |\( \Delta \lambda^\dagger \)| is not necessarily associated
with a smaller cyclic shift offset $|\Delta \kappa|$. Therefore sequences separated by large shift offset $|\Delta \kappa|$ between them may not be safe against small frequency offset. Increasing the minimum separation between sequences therefore does not necessarily improve a sequence set’s overall immunity to frequency offset.

It is worth noting that the frequency of an access device is synchronized to the access point via the downlink synchronization channel, the Doppler frequency is $222 \text{ Hz}$. Since the frequency synthesizer error is typically within the range of 0.1 ppm or $0.0001$, the frequency offset on the uplink is thus determined by the access device frequency synthesizer error plus Doppler frequency. The frequency synthesizer error is typically within the range of 0.1 ppm or $0.0001$, giving rise to a total of $644 \text{ Hz}$ frequency offset at the access point receiver, which is $444 \text{ Hz}$, giving rise to a total of $644 \text{ Hz}$ frequency offset at the access point. The subcarrier spacing of the LTE random access channel (RACH) is $1.25 \text{ kHz}$. This corresponds to a $|\Delta \lambda|$ value of $0.5$. For higher speed scenarios, e.g., high-speed trains ($\sim 300 \text{ km/h}$), and at higher frequency bands (e.g., 3-5 GHz), the frequency offset can be much larger.

**IV. Effect of Frequency Offset on Sequence Detection**

In this section, we study the effect of the frequency offset on the detection performance of a ZC sequence set. From (22), the output of the correlator $x_\ell$ ($\forall \ell \in K$) is

$$z_\ell = \sum_{\kappa \in K'} h_\kappa \gamma (\Delta \lambda_\kappa - \mu \Delta \kappa_\kappa) + \nu_\ell,$$

(30)

where $\Delta \kappa_\kappa$ is the shift offset of $x_\kappa$ with respect to $x_\ell$. As $z_\ell$ is a linear combination of Gaussian variables, \(\{h_\kappa, \kappa \in K'\}\), it’s thus also Gaussian, given that the frequency offset of the active sequence $x_\kappa \in K'$ is $\Delta \lambda_\kappa$, i.e., $z_\ell \sim CN\left(0, \sum_{\kappa \in K'} \alpha_\kappa^2 |\gamma (\Delta \lambda_\kappa - \mu \Delta \kappa_\kappa)|^2 + \sigma_\ell^2 \right)$. Hence $\zeta_\ell = |z_\ell|^2$ is exponentially distributed with a mean of

$$\eta_\ell = \sum_{\kappa \in K'} \alpha_\kappa^2 |\gamma (\Delta \lambda_\kappa - \mu \Delta \kappa_\kappa)|^2 + \sigma_\ell^2,$$

(31)

and a variance of $\sigma_\ell^2 = \eta_\ell^2$.

When sequence $x_\ell$ is inactive ($\ell \notin K'$ or $\Delta \kappa_\kappa \neq 0$), i.e., $x_\ell$ is absent from the received signal $y$, (31) represents the mean energy of the total interference (plus noise) at the output of the correlator of $x_\ell$ from all the active sequences. In this scenario, we define (32) to include all possible non-zero ($\kappa \neq \ell$) inter-sequence shift offsets of $K$. Clearly, $|\Delta K| = |K|(|K| - 1)$.

We ignore the thermal noise $\nu_\ell$ at the receiver and focus on the interference from the other sequences, i.e.,

$$\eta_{\ell, K'} = \sum_{\kappa \in K', \kappa \neq \ell} \alpha_\kappa^2 |\gamma (\Delta \lambda_\kappa - \mu \Delta \kappa_\kappa)|^2.$$

(33)

It is clear that when $\Delta \lambda_\kappa = 0$ ($\kappa \in K'$), $\eta_{\ell, K'} = 0$, i.e., the output of the correlator of $x_\ell$ contains no interference from the active sequences. Otherwise, $\eta_{\ell, K'} \neq 0$, which incurs false

$$\Delta K = \left\{ \Delta \kappa_\kappa = \kappa - \ell + lN, \ l = \arg\min_{l \in \mathbb{Z}} |\kappa - \ell + lN|, \ \kappa, \ell \in K, \kappa \neq \ell \right\}$$

(32)
detections of sequence $x_{k'}$ with the probability of
\[ p = e^{-\frac{T}{N_p}}, \]  
\[ T_{k,k'} = \eta_{k,k'} \ln p^{-1}. \] \[ (35) \]

That is, the threshold is $\ln p^{-1}$ times higher than the total interference from the $|K|'$ active sequences. For example, to maintain a false alarm rate of $p = 0.001$ for each sequence detector, $T$ has to be $\sim 8$ dB above the interference.

Consider a case where one of the transmitters of the sequences, e.g., $x_{k'}$ is close to the receiver, i.e., a large $\alpha_k^2$. A high detection threshold $T$ is needed in order to maintain a reasonably low false alarm rate for the detection of other signals. However, this may block weak signals from being detected from the transmitters that are far away from the receiver and are already transmitting at their maximum power. To minimize this near-far effect, the transmit power of a random access signal must be carefully managed.

In LTE, the transmitter first transmits the signal with the minimum transmit power and gradually increases its transmit power at each failed attempt in order to find the power just enough to compensate for the path-loss to avoid blanking out other access signals [16], [33]. This power ramping procedure in effect results in $\alpha_k^2 = 1$ in (33) to yield
\[ \eta_{k,k'} = \sum_{\kappa \in K'} |\gamma(\Delta \lambda_{\kappa} - \mu \Delta K_{\kappa})|^2. \] \[ (36) \]

Taking the mean of $|\gamma(\Delta \lambda_{\kappa} - \mu \Delta K_{\kappa})|^2$ with respect to $\Delta \lambda_{\kappa}$ gives
\[ |\gamma_{\Delta K_{\kappa}}|^2 \triangleq E_{\Delta \lambda_{\kappa}} \left\{ |\gamma(\Delta \lambda_{\kappa} - \mu \Delta K_{\kappa})|^2 \right\} \]
\[ = \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{q=-n}^{N-n-1} e^{j2\pi \mu \Delta K_{\kappa} q \sin \frac{2\pi N p q}{N}} \] \[ (37) \]

assuming $\Delta \lambda_{\kappa}$ is uniformly distributed in $(-\rho, \rho)$, $\forall \kappa \in K$. Substituting (37) into (36) results in
\[ \eta_{k,k'} = \sum_{\kappa \in K'} \max_{\Delta K_{\kappa} \neq 0} |\gamma_{\Delta K_{\kappa}}|^2. \] \[ (38) \]

Assuming the average number of active sequences from $K$ is $M$, the average interference detected at the output at an inactive sequence detector is
\[ \eta = M |\gamma|^2, \] \[ (39) \]

where
\[ |\gamma|^2 \triangleq \frac{1}{|K|} \sum_{\ell \in K} \frac{1}{|K| - 1} \sum_{\ell + \Delta K_{\kappa} \in K} |\gamma_{\Delta K_{\kappa}}|^2 \]
\[ = \frac{1}{|K|(|K| - 1)} \sum_{\ell \in K} \sum_{\ell + \Delta K_{\kappa} \in K} |\gamma_{\Delta K_{\kappa}}|^2 \] \[ (40) \]

is the average interference per sequence of $K$.

It is also beneficial to have the knowledge on the largest average interference per sequence of a given sequence set for the purpose of performance evaluation of a sequence set. Noting that $|\gamma(\Delta \lambda_{\ell + \Delta \kappa} - \mu \Delta K_{\kappa})|^2 \geq 0$, we have (41) to indicate the maximum amount of the average interference that

\[ |\gamma|^2_{\max} = \frac{1}{|K|} \sum_{\ell \in K} \frac{1}{|K| - 1} \sum_{\ell + \Delta K_{\kappa} \in K} \max_{|\Delta \lambda_{\ell + \Delta \kappa} |< \rho} |\gamma(\Delta \lambda_{\ell + \Delta \kappa} - \mu \Delta K_{\kappa})|^2 \]
\[ = \frac{1}{|K|(|K| - 1)} \sum_{\ell \in K} \sum_{\ell + \Delta K_{\kappa} \in K} \max_{|\Delta \lambda_{\ell} |< \rho} |\gamma(\Delta \lambda_{\ell} - \mu \Delta K_{\kappa})|^2 \] \[ (41) \]

\[ = \frac{1}{|K|} \sum_{\Delta K_{\kappa} \in \Delta K} \max_{|\Delta \lambda_{\ell} |< \rho} |\gamma(\Delta \lambda_{\ell} - \mu \Delta K_{\kappa})|^2 \]
\[ = \frac{1}{|\Lambda^\perp_K|} \sum_{\Delta \lambda_{\ell} \in \Lambda^\perp_K} \max_{|\Delta \lambda_{\ell} |< \rho} |\gamma(\Delta \lambda_{\ell} - \mu \Delta K_{\kappa})|^2 \]

\[ \approx \frac{1}{|\Lambda^\perp_K|} \sum_{\Delta \lambda_{\ell} \in \Lambda^\perp_K} \left| \sum_{\Delta \lambda_{\ell} \neq |\Lambda^\perp_K|} \right| d \left| |\Delta \lambda_{\ell} | - |\Lambda^\perp_K| \right| + N^2 \sum_{\Delta \lambda_{\ell} \in \Lambda^\perp_K} d \left( |\Lambda^\perp_K| - 1 \right). \] \[ (44) \]
a sequence in \( K \) can possibly contribute to the false detection of another sequence in \( K \).

For low frequency offset scenarios where the frequency offset is mainly the frequency synchronization error, which is typically in the range of 0.1 ppm [32], or 200 Hz at 2 GHz carrier frequency, i.e., \(|\Delta \lambda| < 0.16\), (41) can then be approximated as

\[
|\gamma|_{\text{max}}^2 \approx \frac{1}{|\Lambda_K|} \sum_{|\Delta \lambda^\dagger| \in \Lambda_K^\dagger} |\gamma(|\Delta \lambda^\dagger| - 0.16)|^2
\]

(42)

or

\[
|\gamma|_{\text{max}}^2 \approx \frac{1}{|\Lambda_{PN}|^2} \sum_{|\Delta \lambda^\dagger| \in \Lambda_K^\dagger} |\text{dsinc}(|\Delta \lambda^\dagger| - 0.16)|^2.
\]

(43)

For high frequency offset scenarios, assuming \(|\Delta \lambda| < 1\), we have (44) at the bottom of the previous page. Since \(|\Delta \lambda^\dagger| \leq \lfloor N/2 \rfloor \) and noting that larger \(|\Delta \lambda^\dagger| \) corresponds to smaller \(|\gamma(|\Delta \lambda^\dagger| - |\Delta \lambda^\dagger|)|^2\), we thus replace \(|\Delta \lambda^\dagger| \) in (44) with \(|N/2|\) to give

\[
|\gamma|_{\text{max}}^2 \approx \frac{1}{|\Lambda_K^\dagger|} \cdot |\text{dsinc}([N/2] - 0.5)|^2 > 1,
\]

(45)

assuming \( N > 1 \). That is, at a high frequency offset, the maximum inter-sequence interference of a ZC sequence set is always larger than the ideal PN sequence.

V. NUMERICAL AND SIMULATION RESULTS

From the previous analysis, we conclude that the overall performance of a sequence set in the presence of a frequency offset is governed by its critical frequency offset spectrum \( \Lambda_K^\dagger \), determined by the shift offset set \( \Delta K \) of the sequence set. In particular, it is the small critical frequency offsets in \( \Lambda_K^\dagger \) that dominate the performance of a sequence set. In order for a sequence set to have a certain degree of frequency offset immunity, it is desirable that it be structured in a way such that any sequence pair whose shift offset corresponds to a small critical frequency offset be excluded from the set. We take a ZC sequence set with \( \mu = 688 \) as a toy example to illustrate this key concept.

The example sequence set defined by \( K \) is of sequence length \( N = 839 \) and initial set size \(|K|=10\) with total \(|\Delta K| = 10 \times (10 - 1) = 90\) non-zero shift offsets. The spectrum of this initial sequence set is plotted in Fig. 4 (a). Since the spectrum is symmetric, we will focus on the positive half (\( \Delta \lambda^\dagger > 0 \)) of the spectrum. Note that since the frequency offset property is dominated by the smallest critical frequency offsets (corresponding to the lower end of the spectrum), the critical frequency offset is plotted in logarithmic scale in the spectrum. There are three \( \Delta_k^\dagger \)'s in \( \Delta K \) with value 50 (i.e., three sequence pairs in the sequence set with an inter-sequence shift offset of 50), corresponding to a critical frequency offset value of one (i.e., \( \Delta \lambda^\dagger = 1 \)); One \( \Delta_k^\dagger = 100 \) in \( \Delta K \) corresponds to a critical frequency offset value of two (i.e., \( \Delta \lambda^\dagger = 2 \)). The correlation between two sequences (normalized to the ideal PN correlation) in the example sequence set is plotted in Fig. 5. As shown in Fig.

Fig. 4. The spectra of the example sequence set. (a) The initial spectrum of the example sequence set; (b) The reshaped spectrum after removal of \(|\Delta \lambda^\dagger| = 1 \); (c) The reshaped spectrum after removal of \(|\Delta \lambda^\dagger| = 2 \); (d) The reshaped spectrum after removal of \(|\Delta \lambda^\dagger| = 9, 25 \); (e) The reshaped spectrum after removal of \(|\Delta \lambda^\dagger| = 16, 105, 121 \).
Fig. 5. The correlation between two sequences in the example sequence set $K$ of length $N = 839$ and root $\mu = 688$. The correlation is normalized to that of an ideal PN.

Fig. 6. Average interference per sequence from both analytical (hollow red, from (40) and (43)) and simulation results (solid red) for $\mu = 688$ and $|\Delta \lambda| < 0.16$ (the low frequency offset scenario). The elimination of certain critical frequency offsets changes the interference. The result from PN simulation (green) is also shown. The PN sequence length is selected to be the same as the ZC sequence, i.e., $N = 839$. The natural period of the PN sequence is $2^{25} - 1$. The average interference for ZC sequences with mixed roots (each ZC sequence in the set has a unique root) is also shown.

Fig. 7. Average interference per sequence from both analytical (hollow red, from (40) and (44)) and simulation results (solid red) for $\mu = 688$ and $|\Delta \lambda| < 1$ (the high frequency offset scenario). The elimination of certain critical frequency offsets changes the interference. The result from PN simulation (green) is also shown. The PN sequence length is selected to be the same as the ZC sequence, i.e., $N = 839$. The natural period of the PN sequence is $2^{25} - 1$. The average interference for ZC sequences with mixed roots (each ZC sequence in the set has a unique root) is also shown.

6 and Fig. 7, the average interference per sequence from (40) is $\sim 26$ dB for low frequency offset scenarios and $\sim 41$ dB for high frequency offset scenarios (relative to the ideal PN correlation). The average interference of the sequence set in the low frequency offset scenario is close to the truncated PN sequence (839 from a natural period of $2^{25} - 1$) and is larger than the truncated PN sequence in the high frequency scenario. In addition, unlike ZC sequences, the inter-sequence interference of the truncated PN sequences is less sensitive to the frequency offset.

Fig. 4 (b) depicts the reshaped spectrum by the removal of the sequences associated with $|\Delta \lambda| = 1$. There are $|K| = 7$ remaining sequences in the reduced sequence set, of which one has shift offsets of $\Delta \kappa = 100$ that corresponds to $\Delta \lambda = 2$. The average interference of a sequence reduces to $\sim 19$ dB and $\sim 28$ dB (cf. Fig. 6 and Fig. 7), respectively. The exclusion of the sequence associated with $\Delta \lambda = 2$ in the spectrum results in a new spectrum as shown in Fig. 4 (c) and a sequence set of size $|K| = 6$. As a result, the mean value of the interference for low and high frequency scenarios drops to $\sim 8$ dB and $\sim 16$ dB, respectively. Further removal of the sequences associated $\Delta \lambda = 9$ in the spectrum along with $\Delta \lambda = 25$ prunes the spectrum as shown in Fig. 4 (d) and further lowers the average interference of a sequence to $\sim 4$ dB and $\sim 12$ dB. The remaining number of sequences is 5. Finally, elimination of the sequences associated with $\Delta \lambda = 16, 105, 121$, etc., leads to the new spectrum as shown in Fig. 4 (e) and the mean interference of less than 0 dB, i.e., less than the corresponding ideal PN sequences, however, at the cost of less available sequences (i.e., $|K| = 3$ available sequences). There is a
tradeoff between the average interference per sequence and the number of available sequences of a sequence set.

We apply (40), (43) and (44) to the sequence sets specified by LTE. In LTE sequence sets used for random access are categorized into low Doppler and high Doppler sets. Low Doppler sets are targeted for use in low frequency offset deployments where low mobility is expected and the frequency offset is mainly composed of synchronization errors. On the other hand, the high Doppler sets are designed for high frequency offset environments where high mobility is expected and the frequency offset may contain high Doppler frequency spread in addition to the synchronization errors.

Fig. 8 and Fig. 9 plot the average interference per sequence of the respective sequence sets designed for low and high frequency offset applications as specified in LTE [22] from (40), (43), (44) as well as simulations. The corresponding spectra of these sequence sets are plotted in Fig. 10.

It is seen that the spectra of the sequence sets for high frequency offset scenarios have much less number of small critical frequency offset components than the sets for low frequency scenarios in order to provide more immunity to frequency offset. Consequently, the total number of available sequences in each set for high frequency offset scenarios is also much less. Sequence sets C and C′ have the least spectral components in the low critical frequency offset range, hence the least average interference in their respective categories as can be seen in Fig. 8 and Fig. 9.

Although the spectrum-pruned ZC sequence sets have less interference compared to the truncated PN sequence, ZC sequence sets have much less number of available sequences, especially for the high frequency offset case, as compared to the truncated PN sequence that can provide as many as $\frac{25}{176}$ sequences (assuming that the minimum inter-sequence shift offset is 512). Since the length of a PN sequence can be selected to be less than its natural period and still maintain a good cross-correlation between sequences, PN sequence-based random access signals have the luxury to generate an arbitrary number of random access signals whose respective sequences are truncated from a PN sequence with an arbitrarily long natural period. Unfortunately, the length of ZC sequences has to be the same as the natural period to maintain good orthogonality. As a remedy, the ZC sequences from different roots have to be added to support the simultaneous access devices, resulting in degraded overall performance. A good compromise has to be made between the size of the access sequences and the overall performance.

Fig. 11 plots the detection performance of ZC sequences in a practical random access cellular environment. In the simulation, a cell supports a maximum number of 64 sequences as random access signals. The roots used to generate the sequences are shown in Table I. For high frequency offset scenarios, the number of sequences generated from the same root is short of 64. Sequences from multiple roots are therefore needed to provide sufficient number of sequences. The number of simultaneous access devices obeyed Poisson distribution with a mean arrival rate of 250 devices/second/cell [33], [34]. Access devices were randomly (uniformly) dropped in the cell and their propagation delays were determined by the distances between the access devices and the access point. The path loss obeyed the COST231-Hata model. The cell radii in the simulations were 790 m and 1080 m. The maximum number of times of power ramping was three. The initial transmit power was decided by the distance along with the initial received target SNR (18 dB). The detection thresholds were set such that the false alarm rate was maintained at 0.001. We see that the overall detection performance of the ZC sequence sets with mixed roots (i.e., Sets $F = C \cup D$, $G = B \cup D$, and $I = A' \cup B' \cup C' \cup D'$) is deteriorated as compared to the expected individual performance provided in Fig. 8 and Fig. 9.

VI. CONCLUSION

A ZC sequence is commonly known to have better autocorrelation properties than a PN sequence. However, it is in general not true in practical applications where a frequency offset is present between the transmitter and the receiver. That is, the perfect autocorrelation property is lost under non-zero frequency offset and the interference between the sequences generated using the autocorrelation property and cyclic shifts can be worse than the PN sequence if the sequence set is not appropriately structured. It is thus desirable to understand the behavior and performance of ZC sequences when applied
Fig. 10. The spectra of the random access sequence sets specified by LTE for low (left column) and high (right column) frequency offset applications. The spectra in the right column contain much fewer number of small critical frequency offset components than the ones in the left column. Consequently, the right column ones are more frequency offset resilient than the left ones, however, the total number of available sequences is also much less.
to practical communication systems. We present an analytical framework to provide insight into the ZC sequence structure and its fundamental limitations in the presence of frequency offset. We propose the concepts of critical frequency offset and its fundamental limitations in the presence of frequency offset. We propose the concepts of critical frequency offset and related sequences,” Proc. IEEE, vol. 68, pp. 593–619, May 1980.


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