Collimation effect inside complete bandgap of electromagnetic surface resonance states on a metal plate perforated with a triangular array of air holes

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Abstract: In this paper the transmission properties of a metal plate perforated with a triangular array of air holes is investigated. We find that the normalized transmittivity exceeds unity within a certain frequency range under normal incidence of a Gaussian beam. Calculations and experiments indicate that the phenomenon results from the collimation effect which only occurs inside the complete bandgap of surface resonance states on the perforated metal plate. The findings present a simple approach for beam collimation.

OCIS codes: (240.6680) Surface plasmons; (160.3918) Metamaterials; (050.1950) Diffraction gratings.

References and links

1. Introduction

It has been reported that a metal film perforated with a periodic array of subwavelength holes can exhibit extraordinary optical transmission (EOT) [1, 2]. The excitation of electromagnetic (EM) surface resonance states (SRSs) bound to the metal-dielectric interface plays an important role for the formation of the remarkable phenomena [1–7]. When such a SRS is excited, the enhanced EM local fields have strongest field strength at the interface, which decays exponentially with increasing distance into both of the neighboring media. These SRSs are termed as surface plasmon polaritons (SPPs) [8, 9] at visible and near-infrared frequencies. With high spatial frequency and enhanced local fields, SPPs have been utilized in photonic sensing for surface morphology and chemical analyses [10], plasmon enhanced photoluminescence [11], and plasmon assisted nanolithography [12]. At lower frequencies, such as in the microwave regime, although metals behave as perfect electric conductors (PECs), a structured perfect metal surface can support spoof surface plasmons [13, 14] with the assistance of localized surface resonance modes [15, 16], and the EOT still exists [17–19]. And the surface-plasmon-like properties of SRSs also have great application potentials in different frequency domains.

Like what have been done to photonic bandgap materials, bandgap engineering of SRSs [20, 21] is very crucial to plasmonic materials or metamaterials for the purpose of light manipulation [22–25]. Notably, the dispersive and dissipative nature of metals in the optical region makes against the attempts to accurately characterize the dispersion and bandgap of SPPs. The first observation of a full bandgap for transverse magnetic (TM) polarized surface modes was reported in 1996 by S. C. Kitson and his associates [26]. They measured reflection spectra from a silver film deposited with a triangular array of dielectric scatterers under TM polarized excitation and observed a full SPP bandgap in the visible spectrum. It is worth
noting that a structured metal film also supports SPPs or SRSs in transverse electric (TE) polarization. A complete bandgap, for all polarizations with respect to all Bloch wave vectors, deserves more attention.

In this paper, we investigate the EM transmission properties of a metal plate which is periodically perforated with a triangular array of air holes. We find by theoretical calculations and microwave experiments that, within a certain frequency range, the transmitted beam is highly collimated under normal incidence of a Gaussian beam. Calculations with modal expansion method (MEM) [27–31] indicate that, for our model plate, there exists a complete SRSs bandgap for all polarizations at 10.20-10.94GHz that is precisely superposed on the frequency range where beam collimation happens. For comparison, we also investigate the dispersion of SRSs on a metal plate perforated with a square array of holes. Calculations indicate that no complete SRSs bandgap and beam collimation can be found in such system. The results imply that the triangular lattice of air holes is instrumental for the complete SRSs bandgap which gives rise to the collimation effect for a normally incident Gaussian beam. The findings present a simple scheme for beam collimation.

2. Model and analysis method

Our sample is fabricated on an aluminum plate which has a thickness of \( t = 2\text{mm} \) and a lateral size of \( 1000\text{mm} \times 1000\text{mm} \). The lattice constant of the triangular hole array is \( p = 30\text{mm} \). The diameter of holes is \( d = 15\text{mm} \). The insert of Fig. 1 schematically illustrates the front surface of the sample plate in the \( xy \) plane at \( z = 0 \) as well as the irreducible Brillouin zone of the triangular lattice. The primitive lattice vectors are \( \vec{a}_1 = p \left( \frac{1}{2} \hat{e}_x + \frac{\sqrt{3}}{2} \hat{e}_y \right) \) and \( \vec{a}_2 = p \left( \frac{1}{2} \hat{e}_x - \frac{\sqrt{3}}{2} \hat{e}_y \right) \), and the corresponding reciprocal lattice vectors are \( \vec{b}_1 = \frac{2\pi}{p} (\hat{e}_x + \frac{1}{\sqrt{3}} \hat{e}_y) \) and \( \vec{b}_2 = \frac{2\pi}{p} (\hat{e}_x - \frac{1}{\sqrt{3}} \hat{e}_y) \) where \( \hat{e}_x \) and \( \hat{e}_y \) denote the unit vectors along \( x \) and \( y \) directions in the Cartesian coordinate.

With the assumption of PEC for metals in our model system, the EM fields in region II \((-t < z < 0)\), containing the sample plate, only exist in the air holes. As such they can be decomposed with a series of guided modes [32]. We note that only those TE\(_{p,q}\) guided modes can be excited by the plane wave incidence, as such the electric field inside the air holes can be written as the superposition of all the forward and backward TE\(_{p,q}\) modes

\[
\vec{E}^\text{II} = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} (a_{p,q} e^{-i\beta_{p,q} z} - b_{p,q} e^{i\beta_{p,q} z}) \vec{g}_{p,q}(x,y),
\]

where the integer pair \((p,q)\) in the subscript denotes a certain mode number \((p=0,1,2,...; q=1,2,3,...)\), i.e. the \((p,q)\)th order of the guided mode, \(a_{p,q}\) and \(b_{p,q}\) are the \((p,q)\)th order coefficients of forward and backward guided waves, \(\beta_{p,q}\) is the wavevector component along \(z\) axis with \(\beta_{p,q} = \sqrt{k_0^2 - T_{p,q}^2}\) and \(T_{p,q}\) being its corresponding in-plane wavevector component. \(k_0\) is the wavevector in vacuum. \(T_{p,q}\) is the \(q\)th root of eigen function \(J'_1(T_{p,q}d/2)/k_0\). \(\vec{g}_{p,q}(x,y)\) is the \((p,q)\)th in-plane mode function of metallic aperture which can be expressed with \(\vec{g}_{p,q}(x,y) = \vec{g}_{p,q}(\rho,\phi)\hat{e}_\rho + \vec{g}_{p,q}(\rho,\phi)\hat{e}_\phi \) in cylindrical coordinates where
\[ g_{\rho,\phi}(\rho, \phi) = N_p'(T_{p,\rho}d / 2)J_p(T_{p,\rho}\rho)\cos(p\phi), \] (2)

\[ g_{\rho,\phi}(\rho, \phi) = -\frac{p}{\rho}N_p'(T_{p,\rho}d / 2)J_p(T_{p,\rho}\rho)\sin(p\phi). \] (3)

\( \hat{e}_r \) and \( \hat{e}_\phi \) donate the unit vectors along radial and tangential directions. \( J_p \) and \( N_p \) are the \( p^{th} \) order Bessel function and Newman function, respectively, while \( J'_p \) and \( N'_p \) represent the derivatives of \( J_p \) and \( N_p \). The magnetic fields can be easily derived from the representation in Eq. (1) of electric fields, according to the Maxwell equations, as:

\[ \vec{H}^n = \sum_{p,q=0}^{\infty} \left( a_{p,q} e^{-i\beta_{p,q}z} - b_{p,q} e^{i\beta_{p,q}z} \right) \vec{g}_{p,q}, \]

\[ = \left\{ \frac{1}{\rho} \frac{\partial g_{\rho,\phi}(\rho, \phi)}{\partial \rho} \hat{e}_\rho + \frac{1}{\rho} \frac{\partial g_{\rho,\phi}(\rho, \phi)}{\partial \phi} \hat{e}_\phi + \left[ \frac{\partial g_{\rho,\phi}(\rho, \phi)}{\partial \rho} - \frac{\partial g_{\rho,\phi}(\rho, \phi)}{\partial \phi} \right] \hat{e}_z \right\}, \] (4)

where \( \hat{e}_z \) is the unit vector along \( z \) direction.

The EM fields in region I (\( z > 0 \)) at the incident side and region III (\( z < -t \)) at the outgoing side of the plate can be expanded with the superposition of Bloch waves,

\[ \vec{E}^I = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left( I_s \delta_{m,n} + r_{m,n} \right) \tilde{P}_{m,n,0}(x,y) e^{-i(k_x \hat{e}_x + k_y \hat{e}_y + k_z \hat{e}_z)}, \] (5)

\[ \vec{E}^III = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{r=0,1} t_{m,n,r}(x,y) e^{-i(k_x \hat{e}_x + k_y \hat{e}_y + k_z \hat{e}_z)}, \] (6)

where \( \tilde{P}_{m,n,0}(x,y) = \frac{1}{k_{x,n}^2 + k_{y,n}^2} \hat{e}_x + \frac{1}{k_{z,n}^2 + k_{z,n}^2} \hat{e}_z \) and \( \tilde{P}_{m,n,1}(x,y) = \frac{k_{x,n}}{k_{x,n}^2 + k_{y,n}^2} \hat{e}_x + \frac{k_{z,n}}{k_{z,n}^2 + k_{z,n}^2} \hat{e}_z \) are the unit vectors of electric fields at local point \( (x, y, z) \) for the \( x \) - and \( y \) -polarized plane waves. \( k_{x,n} \), \( k_{y,n} \) and \( k_{z,n} \) are the \( x \), \( y \) and \( z \) components of the wavevector. \( I_s \) and \( \vec{k}_n \) are the coefficient and the in-plane wavevector component of incident plane wave with \( k_{z,n} = \sqrt{k_{x,n}^2 + k_{y,n}^2} \). \( r_{m,n,r} \) and \( t_{m,n,r} \) are the coefficients of the \( (m,n)^{th} \) reflected and transmitted Bloch waves. \( \vec{G}_{m,n} = m\hat{b}_x + n\hat{b}_y \) is the Bloch wavevector of triangular lattice. By applying the boundary continuum conditions at the two interfaces \( z = 0 \) and \( z = -t \) for the tangential components of EM fields, we derive the coefficients of the forward and backward guided waves within the air holes of the structured layer, and those of the reflected and transmitted Bloch waves in free space.

It is worth noting that, the in-plane field distributions are primarily determined by \( TE_{11} \) guided mode, as the frequency range of our interest is much lower than the cutoff frequency of any higher order guided mode confined in the air holes. And a single-mode expansion is accurate enough to deal with our model system semi-analytically [31].

3. Complete bandgap and beam collimation

We perform transmission measurements in microwave chamber with a pair of directive horn antennas working at 8.2-12.4GHz with a gain factor of 24.8dB. The two antennas are positioned about 6 meters apart from each other, one as emitter and the other as receiver. The
sample plate is on a rotary table in the middle between the horn antennas. The rotary table, driven by computer, gives rise to a finest resolution of 0.1° to the incident angle. Angle-resolved transmittivity $T$ can also be utilized to measure the SRSs dispersion of a perforated metal plate, in that a peak frequency refers to the excitation of a certain SRS, and the corresponding in-plane wavevector can be defined by the polar angle $\theta$ and the azimuthal angle $\phi$ as

$$k_x = \frac{2\pi}{c_0} f \sin \theta \left( \cos \phi \hat{e}_x + \sin \phi \hat{e}_y \right),$$

where $c_0$ refers to the velocity of light in vacuum. The transmittivity is normalized to the wave energy transmitted between the two horn antennas without the sample plate.

Figure 1. Computed (red solid line for MEM calculation, blue dash line for FDTD simulation) transmittivity as a function of frequency under plane wave normal incidence with the electric field $\vec{E}$ along $x$ axis ($\vec{E} \parallel \hat{e}_x$) and measured (black circular dots) transmittivity for the $x$-polarized Gaussian beam normally incident on the sample plate. Two insets present the sketch map of the front surface of the sample (right side) and the irreducible Brillouin zone of the lattice (left side).

Solid and dash lines in Fig. 1 present the transmission spectra computed with MEM and finite-difference-in-time-domain (FDTD) simulation respectively, under normal incidence of $x$-polarized plane wave ($\vec{E} \parallel \hat{e}_x$). We see that the two curves are in good agreement with each other. The measured transmission spectra using the aforementioned horn antennas are also illustrated in Fig. 1 as black circular dots. We see that, the lineshape of measured spectra bears a resemblance with the calculated ones except the hump within the frequency span of $f = 10.20\text{GHz}$, where the normalized transmittivity exceeds unity under normal incidence. In principle, SRSs in odd mode can also give rise to perfect transmission via the channel of evanescent Bragg-scattering of the subwavelength hole arrays, see for example, the narrow peak at 11.53GHz (red solid line in Fig. 1). However such transmission peak can never get observed in experiments, as their appearance critically requires both the perfect periodicity of the array and the plane wave excitation. We see in Fig. 1 that, the normalized transmittivity reaches a maximum of 157% at the frequency of 10.70GHz. And it becomes even larger, increasing to 221%, when the distance between the horn emitter and the plate is reduced from 3.0m to 1.5m in the experiments. The anomalous hump implies that, in this frequency regime, the horn receiver collects more wave energy from the horn emitter as
compared to that transmitted in free space. A reasonable guess is that beam collimation happens around 10.70GHz. For verification, we measured the radiation patterns of the transmitted beam at 10.70GHz with and without the plate, shown as the solid and dash lines in Fig. 2. The half-power beamwidth of the transmitted beam is reduced to 8.1° in E-plane ($\phi = 0^\circ$) and 5.9° in H-plane ($\phi = 90^\circ$), while it is 22.0° and 20.6°, respectively, for the Gaussian beam from the horn antenna emitted into free space.

![Fig. 2. Measured radiation patterns in (a) E-plane ($\phi = 0^\circ$) and (b) H-plane ($\phi = 90^\circ$) of transmitted beams with (solid lines) and without (dash lines) the sample plate normally incident by a horn antenna.](image-url)
Fig. 3. Spatial distribution of electric fields ($E_x$) in the $xz$ plane (a) in free space at 10.7GHz, (b) through the sample with a triangular array at 10.7GHz, (c) through the sample with a square array at 9.5GHz, calculated by FDTD simulations. The Gaussian beam is normally incident on the sample plate with the electric field $E$ along $x$ axis ($\hat{E} \parallel \hat{x}$).

The beam collimation around 10.70GHz is also numerically verified by FDTD simulations [33]. In the simulation, the model plate has a lateral size of 390mm×363.7mm, the problem domain has a volume of 400mm×400mm×1400mm, and the technique of perfect matched layer (PML) is applied for the boundaries. A one-way Gaussian beam, which is linearly polarized with electric field $E$ along the $x$ axis ($\hat{E} \parallel \hat{x}$), is adopted to be normally incident on the sample plate. The beam waist has a dimension of 50mm×40mm in $xy$ plane, and is 350mm away from the model plate. The color charts in Fig. 3(b) presents the spatial distribution of electric field component $E_x$ at 10.70GHz in the $xz$ plane. It is clearly shown that the transmitted beam becomes highly directional with a nearly flat wavefront as compared to the incident Gaussian beam propagating in free space [see Fig. 3(a)]. The results shown in Figs. 3(a) and 3(b) present an intuitive picture for the origin of the anomalously enhanced transmittivity observed in experiments. More calculations show that a metal plate perforated with a square array of air holes cannot collimate Gaussian beam incidence at any frequency with the absence of complete SRSs bandgap (the band structure for these two lattices will be discussed later on). The field distribution at 9.5GHz is shown in Fig. 3(c) in which all the geometric parameters (including plate thickness, lattice constant and hole diameter) are the same as those for the triangular array in Fig. 3(b).
Fig. 4. Calculated (a) and measured (b) transmittivity through the sample plate under TE polarized incidence at $\varphi = 0^\circ$ ($E \parallel \hat{e}_y$) from horn antenna at different incident angles $\theta = 0^\circ, 3^\circ, 6^\circ, 10^\circ, 15^\circ, 22^\circ, 29^\circ$, respectively. (c) and (d) are calculated and measured transmittivity for TE polarized incidence at $\varphi = 90^\circ$ ($E \parallel \hat{e}_x$). Gray dash-dot lines depict the resonance states on two branches with respect to the incident angle, or equivalently speaking the in-plane wavevector.

Angle-dependent transmission spectra are heuristic for us to have a better understanding on the phenomena, since the spectra provide us a direct way to characterize the excited SRS branches [see gray dash-dot lines in Fig. 4] by tracing the peak frequencies and the corresponding incident angles. Figures 4(a) and 4(c) show the calculated transmittivity under TE polarized incidence for $\varphi = 0^\circ$ and $\varphi = 90^\circ$ with respect to different incident angles $\theta = 0^\circ, 3^\circ, 6^\circ, 10^\circ, 15^\circ, 22^\circ, 29^\circ$, respectively. Figures 4(b) and 4(d) present the measured results which are consistent with the calculations. We also see from measured transmission spectra that, under normal incidence, an anomalous hump is measured around 10.70GHz, which becomes lower and disappears rapidly at a small incident angle.

Fig. 5. Dispersion diagrams for (a) TE and (b) TM polarized SRSs for the sample with a triangular array in even mode (circular dots) and odd mode (dash lines). Measured transmittivity is plotted in colormap as a function of frequency and in-plane wavevector.

To get a clear-cut picture on the band structure, we calculate the dispersion of SRSs with MEM by solving the minimum determinant of eigen-equations of the SRSs on the plate, as
shown in Fig. 5. An alternative way to calculate the SRSs dispersion is to trace the peak frequencies as a function of in-plane wavevector $k_{||}$. Both methods give the same results. The gray dash lines are the odd-mode SRSs grazing on the periodic surface [28]. We see that the even-mode SRSs, marked by circular dots possess a bandgap at 10.20-11.53GHz under TM-polarized excitation, which fully covers the TE-polarized bandgap at 10.20-10.94GHz. More calculations on dispersion diagram show that, below the Rayleigh frequency $f = \frac{2 \ c_0}{\sqrt{3} \ p} = 11.53GHz$ which is scaled to the array period, there always exists a complete bandgap, wider or narrower, for larger or smaller diameter of air holes. As a function of frequency $f$ and in-plane wavevector $k_{||}$, the measured transmittivity are plotted in colormap in Fig. 5. The region marked by white color, where the normalized transmittivity exceeds unity, implies the happening of beam collimation. We can see that the bright curves fit the calculated branches of the even-mode SRSs very well, and the frequency range where collimation happens falls inside the complete bandgap marked by shadowed region in Fig. 5.

We also calculate the SRSs dispersion of a model plate perforated with a square array of air holes, as shown in Fig. 6. The geometric parameters (including plate thickness, lattice constant and hole diameter) are chosen to be the same as those of the sample plate shown in Fig. 1, for the purpose of a fair comparison. Results shown in Fig. 6 and Fig. 3(c) indicate that no complete bandgap exists in such system, and no collimation effect happens at any frequency. More calculations by sweeping parameters reveal that these conclusions are universal for the metal plate perforated with a square array of air holes. Obviously, the triangular lattice is crucial for the formation of the complete bandgap and the beam collimation effect. To the best of our knowledge, only the TM polarized bandgap was investigated in previous work [26]; no any report has been addressed on the complete SRSs bandgap for all polarizations. As a complete SRSs bandgap suppresses any surface mode that propagates along the interface, the localized modes excited in each air hole cannot be regulated with a well-defined non-zero in-plane wavevector. As a consequence, the wave energy of incident waves is redistributed in a locally resonant manner, without phase information inherited, and radiated out along the direction of surface normal that is perpendicular to the perforated slab, giving rise to the beam collimation effect.

4. Conclusions

In summary, we show that a metal plate perforated with a triangular array of air holes possesses a complete SRSs bandgap for all polarizations, and thus behaves as a beam collimator with high transparency. The normalized transmittivity under normal incidence of Gaussian beam is measured exceeding unity as a result of collimation effect. Further calculations reveal that the triangular lattice of the air holes is crucial for the formation of the

![Fig. 6. Dispersion diagrams for (a) TE and (b) TM polarized SRSs for the sample with a square array in even mode (circular dots) and odd mode (dash lines).](image-url)
complete SRSs bandgap and the beam collimation effect. The findings provide a simple way for beam collimation in Terahertz and other optical frequencies.

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