Modelling and controller design for discrete-time networked control systems with limited channels and data drift

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\textbf{Abstract}

This paper is concerned with modelling and controller design for a discrete-time networked control system with limited channels and data drift. For the networked control system, available communication channels are usually limited. Moreover, the data received by the actuator may be different from the data sent by the controller and this phenomenon is referred to as data drift. By taking limited channels and data drift into account, a new model for the networked control system is established. Then a new Lyapunov functional is constructed to drive some stability criteria. Based on these stability criteria, a channel utilization-based switched controller is designed to asymptotically stabilize the networked system in the sense of mean-square. The proposed design method can enhance robustness of the networked control system to data drift and external disturbances. A numerical example is given to illustrate the effectiveness of the proposed controller design.

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1. Introduction

Networked control systems (NCSs) are spatially distributed systems in which sensors, actuators, and controllers are communicated through a shared band-limited digital communication network. Networked control has received considerable attention in the last decade \cite{1,7,12,15,18,20,24}. Several advantages of the network architectures include reduced system wiring, plug and play devices, increased system agility, and ease of system diagnosis and maintenance. However, due to the introduction of the network, two challenging issues in NCSs are the effects of both network-induced delays and/or data packet dropouts on the system performance since both of them can degrade the performance of control systems designed without considering them and even de-stabilize the systems. These two issues have attracted considerable attention in the literature, see \cite{1,7,12,15,18,20,24}, to mention a few. On the other hand, the sharing of the network by multiple nodes inevitably induces communication constraints such as limited channels, limited data rate and limited data bandwidth. Communication constraints may degrade the performance of control systems, and there are some results to reduce the negative effects of communication constraints on traditional control systems \cite{10,14,17,26}. For NCSs, the effects of communication constraints have been paid attention and several results have been reported. For example, Guo \cite{2} investigated the joint design of dynamic output feedback controller and network access assignment sequences for NCSs equipped with insufficient communication channels. Guo and Jin \cite{3} addressed the integrated design of controller and communication sequences for an NCS under consideration of medium access limitations and measurement quantization. Guo et al. \cite{4} studied the stability analysis and controller design for linear systems involving a network of sensors and actuators, which are triggered in groups by random events. Heemels et al. \cite{6} derived bounds on the maximally allowable transmission interval (MATI) and the

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maximally allowable delay (MAD) that guarantee stability of an NCS in the presence of communication constraints. Klinkheie et al. [9] considered fault-tolerant control for an NCS, where the sensors, controller and actuators were inter-connected via various medium access control protocols. Song et al. [19] studied the $H_\infty$ filtering problem for a class of discrete-time NCSs with communication constraints. Wang and Yang [22] presented robust $H_\infty$ model reference tracking control for an NCS with communication constraints.

For continuous-time NCSs under consideration of communication constraints, several results are mentioned above in [2–4,6,22]. However, these results may not be applicable in discrete-time NCSs. Moreover, the network-induced characteristics such as packet dropouts and network-induced delays are neglected in [9,19]. It is well known that in practical NCSs, the number of available communication channels is usually limited, thus, at each time step, the sensors and controllers can only access a finite amount of communication channels. Throughout this paper, we consider the communication constraint which is induced by limited communication channels in an NCS. For a discrete-time NCS, since the network-induced characteristics such as limited channels, packet dropouts and network-induced delays may degrade the control performance, it is significant to propose an appropriate controller design method to improve the performance of the considered NCS. However, such a problem has been paid little attention in the existing literature, which is the first motivation of the current study.

In NCSs, the occurrence of quantization errors, external disturbances and network noises induces the phenomenon as data drift. For a discrete-time NCS considering packet dropouts and network-induced delays, it is of paramount importance to study how to enhance robustness of the NCS to data drift. However, data drift in an NCS has not been taken into account in the existing literature, which is the second and main motivation of the current study.

By lumping packet dropouts and network-induced delays into one item, Liu and Fridman [11], and Meng et al. [13] studied the stability and stabilization of continuous-time NCSs. Quevedo and Nešić [16] proposed a predictive networked control scheme for a discrete-time nonlinear multiple-input multiple-output (MIMO) plant. Moreover, small time-delays up to a fixed threshold were incorporated in the plant model, and signals, which were delayed more than the threshold, were considered as “lost” in [16]. Packet dropouts and network-induced delays were lumped into one item in Xiong and Lam [25] to study stabilization of discrete-time NCSs. Adopting the lump sum of packet dropouts and network-induced delays simplifies the controller design to a certain extent. However, adopting such a lump sum also induces some difficulty for distinguishing the effects of packet dropouts from the effects of network-induced delays on the stabilization of the considered NCS. So it is interesting to consider packet dropouts and network-induced delays in a discrete-time NCS separately. The non-uniform distribution characteristic of network-induced delays was considered in [21,27]. In practical situations, packet dropouts and network-induced delays in an NCS may be non-uniformly distributed. By taking limited channels and data drift into account, and studying packet dropouts and network-induced delays separately, this paper establishes a new model for a discrete-time NCS. The non-uniform distribution characteristic of packet dropouts and network-induced delays is employed in modelling.

In this paper, we introduce the channel utilization-based switched controllers, and establish a new model for the NCS with both limited channels and controller-to-actuator data drift. Based on the newly established model, we consider the problems of $H_\infty$ performance analysis and controller design. Even for an NCS considering both sensor-to-controller and controller-to-actuator data drift, the proposed modelling and controller design will be still applicable.

The contributions of this paper include:

- Establishing a new model for a discrete-time NCS with both limited channels and controller-to-actuator data drift.
- Introducing the channel utilization-based switched controllers for the considered NCS.
- Deriving a controller design criterion, which can enhance robustness of an NCS to data drift and external disturbances, for an NCS with both limited channels and controller-to-actuator data drift.

The remainder of this paper is organized as follows. Section 2 establishes a new model for a discrete-time NCS with both limited channels and controller-to-actuator data drift. Sections 3 and 4 are concerned with $H_\infty$ performance analysis and controller design, respectively, for the considered NCS. The results of numerical simulation are presented in Section 5. Conclusions are drawn in Section 6.

Notation. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space; $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices. $I$ and $0$ represent an identity matrix and a zero matrix with appropriate dimensions, respectively. $E$ stands for the expectation operation. $*$ denotes the entries of a matrix implied by symmetry. Matrices, if not explicitly stated, are assumed to have appropriate dimensions.

2. Modelling for a discrete-time NCS

Consider a linear time-invariant discrete-time NCS described by
\[
\begin{align*}
\dot{x}_k & = Ax_k + B_1u_k + B_2w_k \\
\dot{z}_k & = Cx_k + Du_k \\
x_k & = \phi_k, \quad k = -L_M, \ldots, 0
\end{align*}
\]  
(1)

where \( x_k \in \mathbb{R}^n, u_k \in \mathbb{R}^n, w_k \in \mathbb{R}^p \) and \( z_k \in \mathbb{R}^q \) are the state vector, control input vector, disturbance input and controlled output, respectively; \( w_k \) is assumed to belong to \( L_2[0, \infty) \); \( \phi(\cdot) \in \mathbb{R}^q \) is a discrete vector valued initial function; \( k \) is an integer and \( k \geq -L_M; A, B_1, B_2, C \) and \( D \) are known constant matrices of appropriate dimensions.

Suppose that the shared communication medium can provide simultaneously \( \delta (1 \leq \delta < m) \) controller-to-actuator communication channels and \( n \) sensor-to-controller communication channels, which implies that the controller-to-actuator communication channels are assumed to be limited. In such an NCS, only \( \delta \) of \( m \) elements of \( u_k \) can be transmitted through the communication channels, while the others are dropped. Let the binary-valued variables \( \delta_k (i = 1, 2, \ldots, m) \) denote the medium access status of the \( i \)th element of \( u_k \), i.e. \( \delta_k : \mathbb{R} \to \{0, 1\} \), where 1 implies “accessing” and 0 implies “not accessing”, and \( \delta_{1k} + \cdots + \delta_{mk} = \delta \). The actuator is chosen to be a zero order holder (ZOH) which stores the recently received control input. In the case that \( \delta_{ik} = 0 \), the actuator ignores the \( i \)th element of \( u_k \) by assuming a zero value.

Let \( d_k \) and \( p_k \) denote the length of the network-induced delays and the number of consecutive packet dropouts at time instant \( k \), respectively. Suppose that \( d_m \leq d_k \leq d_M, p_m \leq p_k \leq p_M \), where \( d_m, d_M, p_m \) and \( p_M \) are given integers. In the case that there exist packet dropouts and network-induced delays, the latest available control inputs are used to control the plant. Based on the above given communication protocol, the control law \( u_k \) for the NCS considering packet dropouts, network-induced delays and controller-to-actuator medium access constraints can be described as

\[
u_k = \bar{W}_{sk}K_s x_k - p_k - d_k
\]  
(2)

where \( \bar{W}_{sk} = \text{diag}(\delta_{1k}, \delta_{2k}, \ldots, \delta_{mk}) \), and \( \delta_{1k} = 1 \) or \( \delta_{1k} = 0 \) \( (i = 1, 2, \ldots, m) \). \( K_s \) denotes the switched controller gain which switches according to \( \bar{W}_{sk} \).

**Remark 1.** For the control law constructed in (2), if \( \bar{W}_{sk} \) switches from one mode to another mode, \( K_s \) switches correspondingly. Moreover, the switching of the controller gain \( K_s \) in this paper is stochastic. Notice that the switched controller gain \( K(M_p, M_n) \) is adopted in [4]. Since the activation status of the \( n \) sensors and the activation status of the \( m \) actuators are modelled by two independent Markov processes, the switching of the controller gain \( K(M_p, M_n) \) in [4] is also governed by Markov processes. Then one can conclude that the switched controller gain \( K_s \) in this paper is different from the switched controller gain \( K(M_p, M_n) \) in [4].

**Remark 2.** Notice that the zero-strategy is adopted in this paper to deal with the problem of limited channels. That is, if \( \delta_{1k} = 0 \), the actuator ignores the \( i \)th element of \( u_k \) by assuming a zero-value. In fact, such a zero-strategy can be extended to the hold-strategy, which means that if \( \delta_{1k} = 0 \), keep the \( i \)th element of the control input stored in the actuator unchanged; if \( \delta_{1k} = 1 \), substitute the \( i \)th element of the control input stored in the actuator with the \( i \)th element of the recently received control input \( u_k \).

Define \( q_l = \frac{p_m}{p_m^2}, \) where \( q_l \) is the largest integer smaller than or equal to \( \frac{p_m}{p_m^2} \). Suppose that packet dropouts are non-uniformly distributed, and the probability of \( p_m \) to \( q_l \) packets dropped is \( \sigma_l \), where \( \sigma_l \in [0, 1] \). Then the probability of \( q_l \) to \( p_m \) packets dropped is \( 1 - \sigma_l \). The above given statistic characteristic can be described by

\[
\begin{align*}
\begin{cases}
\text{Prob}(p_m \leq p_k < q_l) = \sigma_l \\
\text{Prob}(q_l \leq p_k \leq p_m) = 1 - \sigma_l
\end{cases}
\end{align*}
\]  
(3)

Define a stochastic variable \( \sigma_{k1} \)

\[
\sigma_{k1} = \begin{cases} 
1, & p_m \leq p_k < q_l \\
0, & q_l \leq p_k \leq p_m
\end{cases}
\]  
(4)

By using the Bernoulli distributed white sequence to describe the stochastic variable \( \sigma_{k1} \), one has

\[
\begin{align*}
\begin{cases}
\text{Prob}(\sigma_{k1} = 1) = E(\sigma_{k1}) = \sigma_l \\
\text{Prob}(\sigma_{k1} = 0) = 1 - E(\sigma_{k1}) = 1 - \sigma_l
\end{cases}
\end{align*}
\]  
(5)

By taking the non-uniform distribution characteristic of packet dropouts into consideration, the control law in (2) is converted into

\[
u_k = \sigma_{k1} \bar{W}_{sk}K_s x_k - p_{k1} - d_k + (1 - \sigma_{k1}) \bar{W}_{sk}K_s x_k - p_{k2} - d_k
\]  
(6)

where

\[
p_{k1} = \begin{cases} 
p_k, & p_m \leq p_k < q_l \\
p_{k}, & q_l \leq p_k \leq p_m
\end{cases}
\]  
(7)
\[ p_{k_2} = \begin{cases} p_k, & q_1 \leq p_k \leq p_M \\ p_2, & p_m \leq p_k < q_1 \end{cases} \]  
(8)

with \( p_1 \) and \( p_2 \) being constants, and \( p_m \leq p_1 < q_1, q_1 \leq p_2 \leq p_M \).

Combining the system (1) with the control law (6) together, one can establish the model for the discrete-time NCS considering limited channels and non-uniformly distributed packet dropouts.

**Remark 3.** Notice that packet dropouts and network-induced delays are lumped into one item in [25], and such an approach simplifies the controller design to a certain extent. However, adopting the lump sum of packet dropouts and network-induced delays induces some difficulty for distinguishing the effects of packet dropouts from the effects of network-induced delays on the stabilization of the considered NCS. By considering packet dropouts and network-induced delays separately, this paper establishes a new model for the control law \( u_k \) in (6). The established control law in (6) includes the control law in [25] as its special case.

In what follows, we will take the non-uniform distribution characteristic of network-induced delays into account. For this purpose, define \( q_2 = \left\lfloor \frac{d_{m_1} - d_{m_2}}{\Delta} \right\rfloor \), where \( q_2 \) is the largest integer smaller than or equal to \( \frac{d_{m_1} - d_{m_2}}{\Delta} \). The non-uniform distribution characteristic of network-induced delays is described as

\[
\begin{align*}
\text{Prob}\{d_m \leq d_k < q_2\} &= \tilde{\sigma}_2 \\
\text{Prob}\{q_2 \leq d_k \leq d_M\} &= 1 - \tilde{\sigma}_2
\end{align*}
\]  
(9)

where \( \tilde{\sigma}_2 \in [0, 1] \).

By defining a stochastic variable \( \sigma_{k_2} = 1 \) in the case that \( d_m \leq d_k < q_2 \); \( \sigma_{k_2} = 0 \) in the case that \( q_2 \leq d_k \leq d_M \) and constructing a control law which is similar to the control law in (6), one can establish the model for the NCS with limited channels and non-uniformly distributed network-induced delays. The corresponding result is omitted here for briefness.

In the case that both packet dropouts \( p_k \) and network-induced delays \( d_k \) are non-uniformly distributed, and the distribution probabilities of packet dropouts and network-induced delays are different, the control law is described as

\[
u_k = \sigma_{k_1} \sigma_{k_2} \hat{W}_{\Delta_k} K_{\Delta_k} x_{k-p_{k_1}-d_{k_1}} + \sigma_{k_1} (1- \sigma_{k_2}) \hat{W}_{\Delta_k} K_{\Delta_k} x_{k-p_{k_2}-d_{k_2}} + (1- \sigma_{k_1}) \sigma_{k_2} \hat{W}_{\Delta_k} K_{\Delta_k} x_{k-p_{M_2}-d_{M_2}} + (1- \sigma_{k_1}) (1- \sigma_{k_2}) \hat{W}_{\Delta_k} K_{\Delta_k} x_{k-p_{M_2}-d_{M_2}} \]

where the definitions of \( d_{k_1} \) and \( d_{k_2} \) are similar to the definitions of \( p_{k_1} \) and \( p_{k_2} \), respectively.

Since the products of stochastic variables \( \sigma_{k_1} \) and \( \sigma_{k_2} \) are included in the control law (10), it is complicated but possible to study \( H_{\infty} \) performance analysis and controller design for the NCS (1). For brevity, when the non-uniform distribution characteristic of both packet dropouts and network-induced delays is considered, we assume that the variation criteria of packet dropouts and network-induced delays are the same, which implies that \( d_m = p_m, d_M = p_M, \sigma_1 = \sigma_2 \). Under such an assumption, we can lump packet dropouts and network-induced delays into one item. Define \( d_k + p_k = L_k \) and suppose that \( L_m \leq L_k \leq L_M \). By taking the non-uniform distribution characteristic of \( L_k \) into consideration, one can establish a new model for the discrete-time NCS with limited channels.

**Remark 4.** It should be pointed out that although packet dropouts and network-induced delays are lumped into one item \( L_k \) to simplify the controller design, it is still easy to distinguish the effects of packet dropouts from the effects of network-induced delays on the stabilization of the considered NCS since the variation criteria of packet dropouts and network-induced delays are assumed to be the same. If the variation criteria of packet dropouts and network-induced delays are different, one can adopt the control law (10) to separate packet dropouts from network-induced delays.

Define \( \rho = \left\lfloor \frac{L_m - L_M}{\Delta} \right\rfloor \), where \( \rho \) is the largest integer smaller than or equal to \( \frac{L_m - L_M}{\Delta} \). Suppose that the statistic characteristic of \( L_k \) is described as

\[
\begin{align*}
\text{Prob}\{L_m \leq L_k < \rho\} &= \tilde{\lambda} \\
\text{Prob}\{\rho \leq L_k \leq L_M\} &= 1 - \tilde{\lambda}
\end{align*}
\]  
(11)

where \( \tilde{\lambda} \in [0, 1] \).

Define a stochastic variable \( \lambda_k \)

\[
\lambda_k = \begin{cases} 1, & L_m \leq L_k < \rho \\ 0, & \rho \leq L_k \leq L_M \end{cases}
\]  
(12)

By using the Bernoulli distributed white sequence to describe the stochastic variable \( \lambda_k \), one has

\[
\begin{align*}
\text{Prob}\{\lambda_k = 1\} &= E(\lambda_k) = \tilde{\lambda} \\
\text{Prob}\{\lambda_k = 0\} &= 1 - E(\lambda_k) = 1 - \tilde{\lambda}
\end{align*}
\]  
(13)

Taking the non-uniform distribution characteristic of \( L_k \) into consideration, one can construct the following control law

\[
u_k = \lambda_k \hat{W}_{\Delta_k} K_{\Delta_k} x_{k-\lambda_k} + (1- \lambda_k) \hat{W}_{\Delta_k} K_{\Delta_k} x_{k-\lambda_k}
\]

(14)
where
\[
L_{k1} = \begin{cases} 
L_k, & L_m \leq L_k < \rho \\
I_1, & \rho \leq L_k \leq L_M 
\end{cases}
\]
\[
L_{k2} = \begin{cases} 
L_k, & \rho \leq L_k \leq L_M \\
I_2, & L_m \leq L_k < \rho 
\end{cases}
\]
with \(I_1\) and \(I_2\) denoting constants, and \(L_m \leq I_1 < \rho, \rho \leq I_2 \leq L_M\).

Combining the system (1) with the control law (14) together, one has the system
\[
\begin{align*}
\dot{x}_{k+1} &= \psi_{1k} + (\lambda_k - \bar{\lambda})\psi_{2k} + B_2u_k \\
\dot{z}_k &= \psi_{3k} + (\lambda_k - \bar{\lambda})\psi_{4k} \\
\dot{x}_k &= \phi_k, & k = -L_M, \ldots, 0
\end{align*}
\]
where
\[
\begin{align*}
\psi_{1k} &= Ax_k + \bar{\lambda}B_1\tilde{W}_{ik}K_1x_{k-L_{k1}} + (1 - \bar{\lambda})B_1\tilde{W}_{ik}K_2x_{k-L_{k2}} \\
\psi_{2k} &= B_1\tilde{W}_{ik}K_3(x_{k-L_{k1}} - x_{k-L_{k2}}) \\
\psi_{3k} &= Cx_k + \bar{\lambda}D\tilde{W}_{ik}K_1x_{k-L_{k1}} + (1 - \bar{\lambda})D\tilde{W}_{ik}K_2x_{k-L_{k2}} \\
\psi_{4k} &= D\tilde{W}_{ik}K_3(x_{k-L_{k1}} - x_{k-L_{k2}})
\end{align*}
\]

**Remark 5.** The non-uniform distribution characteristic of packet dropouts and network-induced delays is taken into account in (17). In the case that \(\rho = L_M + 1\) and \(\lambda = 1\), the system (17) reduces to the system without considering such a non-uniform distribution characteristic.

When the control inputs are transmitted through network medium, data drift is unavoidable. In what follows, we take the controller-to-actuator data drift into account. To reflect the influences of the controller-to-actuator data drift, suppose that \(\delta_k = y + \bar{d}_k\bar{e} (i = 1, 2, \ldots, m)\) for those \(\delta_k\) satisfying \(\delta_k \neq 0\), where \(y, \bar{d}, \bar{e}\) are given scalars, \(\bar{d}_1^T\bar{e}_k \leq 1\). Under consideration of controller-to-actuator data drift, the control law in (14) is converted into
\[
u_k = \lambda_k(y + \bar{d}_k\bar{e})\tilde{W}_{ik}K_1x_{k-L_{k1}} + (1 - \lambda_k)(y + \bar{d}_k\bar{e})\tilde{W}_{ik}K_2x_{k-L_{k2}}
\]
where \(\lambda_k, \tilde{W}_{ik}, K_1, L_{k1}\) and \(L_{k2}\) are the same as the ones in (14). Then the system (17) is converted into
\[
\begin{align*}
\dot{x}_{k+1} &= \psi_{1k} + (\lambda_k - \bar{\lambda})\psi_{2k} + B_2u_k \\
\dot{z}_k &= \psi_{3k} + (\lambda_k - \bar{\lambda})\psi_{4k} \\
\dot{x}_k &= \phi_k, & k = -L_M, \ldots, 0
\end{align*}
\]
where
\[
\begin{align*}
\psi_{1k} &= Ax_k + \bar{\lambda}(y + \bar{d}_k\bar{e})B_1\tilde{W}_{ik}K_1x_{k-L_{k1}} + (1 - \bar{\lambda})(y + \bar{d}_k\bar{e})B_1\tilde{W}_{ik}K_2x_{k-L_{k2}} \\
\psi_{2k} &= (y + \bar{d}_k\bar{e})B_1\tilde{W}_{ik}K_3(x_{k-L_{k1}} - x_{k-L_{k2}}) \\
\psi_{3k} &= Cx_k + \bar{\lambda}(y + \bar{d}_k\bar{e})D\tilde{W}_{ik}K_1x_{k-L_{k1}} + (1 - \bar{\lambda})(y + \bar{d}_k\bar{e})D\tilde{W}_{ik}K_2x_{k-L_{k2}} \\
\psi_{4k} &= (y + \bar{d}_k\bar{e})D\tilde{W}_{ik}K_3(x_{k-L_{k1}} - x_{k-L_{k2}})
\end{align*}
\]

**Remark 6.** Notice that the norm-bounded uncertainty-based method is adopted in (18) to reflect the influences of controller-to-actuator data drift. One can adopt other methods such as the polytopic uncertainty-based method to reflect the influences of data drift. On the other hand, the above given modelling can be extended to deal with an NCS considering both sensor-to-controller and controller-to-actuator data drift, the corresponding result is omitted here for briefness.

**Remark 7.** For an NCS with limited channels, how to make full use of the communication channels is significant. To guarantee that the performance of the considered NCS satisfies the users’ requirements, one should assign the highest transmission priority to the subsystems which are most likely to be unstable. For the system (19), if setting \(\delta_k = 1 (i = 1, 2, \ldots, m)\) introduces better system performance, one should allocate more transmission priority to the \(i\)th element of \(u_k\); otherwise, one should allocate less transmission priority to the \(i\)th element of \(u_k\).
Definition 1. The steady state solution of the system (17) (or system (19)) is asymptotically stable in the sense of mean-square if \( \lim_{k \to \infty} E||x_k||^2 = 0 \) holds, where \( ||x_k|| \) denotes the Euclidean norm of the vector \( x_k \).

Notice that \( L_m \leq L_k < \rho \), \( \rho \leq L_{k+1} \leq L_M \). Define \( \mathcal{I} = \left[ \frac{\ln \rho}{2}, \frac{\ln L_k}{2} \right], \bar{\mathcal{I}} = \left[ \frac{\ln \rho}{2}, \frac{\ln L_{k+1}}{2} \right] \), where \( \mathcal{I} \) and \( \bar{\mathcal{I}} \) are the largest integers smaller than or equal to \( \frac{\ln \rho}{2} \) and \( \frac{\ln L_k}{2} \) respectively. For convenience of \( H_\infty \) performance analysis and controller design, we introduce the scalars \( v_{k1} \) and \( v_{k2} \), where

\[
\begin{align*}
v_{k1} &= \begin{cases} 1, & L_m \leq L_k < \mathcal{I} \\ 0, & \mathcal{I} \leq L_k < \rho \end{cases} \\
v_{k2} &= \begin{cases} 1, & \rho \leq L_{k+1} < \bar{\mathcal{I}} \\ 0, & \bar{\mathcal{I}} \leq L_{k+1} \leq L_M \end{cases}
\end{align*}
\]

(20)

(21)

Based on the established model (19), we will consider \( H_\infty \) performance analysis and controller design for the NCS with both limited channels and controller-to-actuator data drift.

3. \( H_\infty \) performance analysis

This section is to analyze \( H_\infty \) performance for an NCS with both limited channels and controller-to-actuator data drift. We first consider \( H_\infty \) performance for the NCS with limited channels. Construct the following Lyapunov functional

\[
V(x_k; k) = \sum_{i=1}^{8} V_i(x_k, k)
\]

(22)

where

\[
\begin{align*}
V_1(x_k, k) &= x_k^T P x_k \\
V_2(x_k, k) &= \sum_{i=k-L_m}^{k-1} x_i^T Q_1 x_i + \sum_{i=k-L_m}^{k-1} \sum_{j=k+1}^{k-1} x_i^T Q_1 x_j \\
V_3(x_k, k) &= \sum_{i=k-L_m}^{k-1} x_i^T Q_2 x_i + \sum_{i=k-L_m}^{k-1} \sum_{j=k+1}^{k-1} x_i^T Q_2 x_j \\
V_4(x_k, k) &= \sum_{i=k-L_m}^{k-1} x_i^T Q_3 x_i + \sum_{i=k-L_m}^{k-1} \sum_{j=k+1}^{k-1} x_i^T Q_3 x_j \\
V_5(x_k, k) &= (\rho - L_m) \sum_{i=k-L_m}^{k-1} \sum_{j=k+1}^{k-1} \eta_i^T Z_1 \eta_j \\
V_6(x_k, k) &= (L_m - \rho) \sum_{i=k-L_m}^{k-1} \sum_{j=k+1}^{k-1} \eta_i^T Z_2 \eta_j \\
V_7(x_k, k) &= L_m \sum_{i=k-L_m}^{k-1} \sum_{j=k+1}^{k-1} \eta_i^T Z_3 \eta_j \\
V_8(x_k, k) &= L_m \sum_{i=k-L_m}^{k-1} \sum_{j=k+1}^{k-1} \eta_i^T Z_4 \eta_j
\end{align*}
\]

with \( P, Q_1, Q_2, Q_3, Q_4, Z_1, Z_2, Z_3 \) and \( Z_4 \) denoting symmetric positive definite matrices, and \( \eta_j = x_{i+1} - x_i \).

In order to provide a tighter estimation for some finite-sum terms appearing in the forward difference of the Lyapunov functional (22), we introduce the following new lemma.

Lemma 1. For given scalars \( r_m, \bar{r}, \bar{r}_M \), a time-varying scalar \( r_k \) satisfying \( r_m \leq r_k \leq \bar{r}_M \), and a symmetric positive definite matrix \( Z \), one has

\[
-(r_M - r_m) \sum_{i=k-r_m}^{k-1} \eta_i^T Z \eta_i \leq -\alpha_k \frac{r_M - \bar{r}}{r_m - \bar{r}} x_{k-1}^T Z x_{1k} - x_{1k}^T Z x_{1k} - x_{2k}^T Z x_{2k} - (1 - \alpha_k) \frac{r_0}{r_M - \bar{r}} x_{k-\bar{r}}^T Z x_{2k}
\]

(23)

where

\[
\alpha_k = \begin{cases} 1, & r_m \leq r_k \leq \bar{r} \\ 0, & \bar{r} \leq r_k \leq \bar{r}_M \end{cases}
\]

(24)

and \( \bar{r} = \left\lfloor \frac{\ln \rho}{2} \right\rfloor \) with \( \bar{r} \) denoting the largest integer smaller than or equal to \( \frac{\ln \rho}{2} \). \( \eta_1 = x_{i+1} - x_i, Z_{1k} = x_{k-r_m} - x_{k-r_m}, Z_{2k} = x_{k-r_k} - x_{k-r_k} \).
Proof. See Appendix A.

Employing the Lyapunov functional (22) and applying Lemma 1, we now state and establish the following result.

**Theorem 1.** For given scalars \( \mu, \lambda, L_m, \rho \) and \( \gamma \), and the controller gain \( K_\mu \), the system (17) is asymptotically stable in the sense of mean-square with an \( H_\infty \) norm bound \( \gamma \) if there exist symmetric positive definite matrices \( P, Q_1, Q_2, Q_3, Q_4, Z_1, Z_2, Z_3 \) and \( Z_4 \) such that the following linear matrix inequalities (LMIs) hold for every feasible value of \( W_{ak}, v_{k1} \) and \( v_{k2} \)

\[
\begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} \\
* & \Omega_{22} & 0 \\
* & * & \Omega_{33}
\end{bmatrix} < 0
\]

where \( \Omega_{11} = \Pi + \Xi_2 \)

\[
\Pi = \\
\begin{bmatrix}
\Pi_{11} & 0 & 0 & Z_4 & 0 & Z_3 & 0 \\
* & \Pi_{22} & 0 & Z_1 & Z_4 & 0 & 0 \\
* & * & \Pi_{33} & 0 & Z_2 & Z_2 & 0 \\
* & * & * & \Pi_{44} & 0 & 0 & 0 \\
* & * & * & * & \Pi_{55} & 0 & 0 \\
* & * & * & * & * & \Pi_{66} & 0 \\
* & * & * & * & * & * & -\gamma I
\end{bmatrix}
\]

\[
\begin{align*}
\Pi_{11} &= -P + (\rho - L_m + 1)Q_1 + (L_m - \rho + 1)Q_2 + Q_3 + Q_4 + Q_5 - Z_3 - Z_4 \\
\Pi_{22} &= -Q_1 - 2Z_1, \quad \Pi_{33} = -Q_2 - 2Z_2, \quad \Pi_{44} = -Q_3 - Z_1 - Z_4 \\
\Pi_{55} &= -Q_4 - Z_1 - Z_2, \quad \Pi_{66} = -Q_5 - Z_2 - Z_3 \\
\Xi_2 &= -v_{k3} \frac{L - L_m}{L - L_m} \Phi_1 - (1 - v_{k1})(I - L_m) \Phi_2 - v_{k2} \frac{L_m - \rho}{\rho - \rho} \Phi_3 - (1 - v_{k2}) \frac{L_m - \rho}{\rho - \rho} \Phi_4 \\
\Phi_1 &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\
\Phi_2 &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\
\Phi_3 &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\
\Phi_4 &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]
\end{align*}
\]

and

\[
\hat{\Omega}_{12} = \\
\begin{bmatrix}
A^TP & J_{3a}Z_1 & J_{3a}Z_2 & J_{3a}Z_3 & J_{3a}Z_4 \\
\lambda_{3a}P & \lambda_{3a}Z_1 & \lambda_{3a}Z_2 & \lambda_{3a}Z_3 & \lambda_{3a}Z_4 \\
J_{3a}P & J_{3a}Z_1 & J_{3a}Z_2 & J_{3a}Z_3 & J_{3a}Z_4 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
B_{3a}^TP & B_{3a}^T Z_1 & B_{3a}^T Z_2 & B_{3a}^T Z_3 & B_{3a}^T Z_4
\end{bmatrix}
\]

\[
\hat{\Omega}_{13} = \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & C^T & 0 \\
J_{3a}P & J_{3a}Z_1 & J_{3a}Z_2 & J_{3a}Z_3 & J_{3a}Z_4 & \lambda_{3a} & 0 \\
J_{3a}P & J_{3a}Z_1 & J_{3a}Z_2 & J_{3a}Z_3 & J_{3a}Z_4 & 0 & -\lambda_{3a} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\hat{\Omega}_{22} = \text{diag}(\rho_{x_1}, \rho_{x_2}, \rho_{x_3}, \rho_{x_4}), \quad \hat{\lambda} = \lambda(1 - \lambda) \\
\hat{\Omega}_{33} = \text{diag}(-\lambda^{-1}P_{x_1}, \lambda^{-1}P_{x_2}, \lambda^{-1}P_{x_3}, \lambda^{-1}P_{x_4}, -\gamma I, -\lambda^{-1}I) \\
J_{3a} = (A - I)^T, \quad J_{3a} = K_{0}^T \tilde{W}_{ak} B_{3a}^T, \quad J_{3a} = (1 - \lambda)J_{3a}, \quad J_{3a} = K_{0}^T \tilde{W}_{ak} D^T \\
J_{5a} = -J_{2a}, \quad \rho_{x_1} = -(\rho - L_m - 2)Z_1, \quad \rho_{x_2} = -(L_m - \rho)^{-2}Z_2 \\
\rho_{x_3} = L_m^{-2}Z_3, \quad \rho_{x_4} = L_m^{-2}Z_4
\]
**Proof.** See Appendix B.

**Remark 8.** In [5], a discrete delay decomposition approach was proposed to study the stability of linear retarded and neutral systems considering constant time-delay. We adopt the idea of the delay decomposition approach to deal with time-varying network-induced delays and packet droputs in Theorem 1.

**Remark 9.** Theorem 1 considers only the controller-to-actuator medium access constraints. For an NCS considering both controller-to-actuator and sensor-to-controller medium access constraints, suppose that the shared communication medium can provide simultaneously $\delta$ ($1 \leq \delta < m$) controller-to-actuator communication channels and $\theta$ ($1 \leq \theta < n$) sensor-to-controller communication channels. Then the control law in (14) can be described as $u_k = \tilde{z}_k W_{ak} K_{ao} W_{sk} x_k + (1 - \tilde{z}_k) W_{ak} K_{ao} W_{sk} x_{k-1}$, where $W_{ak} = \text{diag}(\theta_{1k}, \theta_{2k}, \ldots, \theta_{nk})$, and $K_{ao}$ denotes the switched controller gain which switches according to $W_{sk}$ and $W_{ak}$. The binary-valued variables $\theta_{ik}$ ($i = 1, \ldots, n$) denote the medium access status of the $i$th element of $x_k$ at the instant $k$, i.e. $\theta_{ik} : \mathbb{R} \rightarrow \{0, 1\}$, where 1 implies “accessing” and 0 implies “not accessing”, and $\theta_{1k} + \cdots + \theta_{nk} = 0$. Substituting $J_{2k}$ and $J_{4k}$ in (25) with $W_{ak} K_{ao}^T W_{sk} B_1^T$ and $W_{ak} K_{ao}^T W_{sk} D_1^T$, respectively, then we can derive a new stability criterion.

From (B.5) and (B.6), one has $-v_{k1} \tilde{z}_k^{r^T} c_k^T z_1 z_k - (1 - v_{k1}) \tilde{z}_k^{r^T} z_1 z_k < 0$ for $\zeta_{1k} \neq 0$ and $\zeta_{2k} \neq 0$ due to the fact that $Z_1 = Z_1^T > 0$; and $-v_{k2} \rho_{-p} \rho_{+} c_k^T z_3 z_k - (1 - v_{k2}) \rho_{-p} \rho_{+} c_k^T z_3 z_k < 0$ for $\zeta_{3k} \neq 0$ and $\zeta_{4k} \neq 0$ due to the fact that $Z_2 = Z_2^T > 0$. However, in [8,23], similar terms are overly bounded by ‘0’, which introduces conservatism for the obtained stability criteria.

Notice that a new bounding technique, which is proposed in Lemma 1, is employed in Theorem 1 to estimate some finite-sum terms appearing in the forward difference of the chosen Lyapunov functional. If $-v_{k1} \tilde{z}_k^{r^T} c_k^T z_1 z_k - (1 - v_{k1}) \tilde{z}_k^{r^T} z_1 z_k$ in (B.5) and $-v_{k2} \rho_{-p} \rho_{+} c_k^T z_3 z_k - (1 - v_{k2}) \rho_{-p} \rho_{+} c_k^T z_3 z_k$ in (B.6) are overly bounded by ‘0’, we can derive the following result immediately.

**Theorem 2.** For given scalars $\mu, \lambda, l_m, m, \rho$ and $\gamma$, and the controller gain $K_o$, the system (17) is asymptotically stable in the sense of mean-square with an $H_\infty$ norm bound $\gamma$ if there exist symmetric positive definite matrices $P, Q_1, Q_2, Q_3, Q_4, Q_5, Z_1, Z_2, Z_3$ and $Z_4$, such that the following LMIs hold for every feasible value of $W_{sk}$

$$\begin{bmatrix} I & \hat{\Omega}_{12} & \hat{\Omega}_{13} \\ \ast & \hat{\Omega}_{22} & 0 \\ \ast & \ast & \hat{\Omega}_{33} \end{bmatrix} < 0$$

where $II, \hat{\Omega}_{12}, \hat{\Omega}_{13}, \hat{\Omega}_{22}$ and $\hat{\Omega}_{33}$ are the same as the corresponding matrices in (25).

To establish the relationship between Theorems 1 and 2, we present the following result.

**Theorem 3.** Consider the system (17). If the LMIs in (28) are satisfied, then the LMIs in (25) are also satisfied.

**Proof.** Notice that (25) can be written as

$$\begin{bmatrix} I & \hat{\Omega}_{12} & \hat{\Omega}_{13} \\ \ast & \hat{\Omega}_{22} & 0 \\ \ast & \ast & \hat{\Omega}_{33} \end{bmatrix} + \begin{bmatrix} \mathcal{Z}_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0$$

(29)

Considering that $\mathcal{Z}_2 < 0$, one can conclude that if the LMIs in (28) are satisfied, the LMIs in (29) are also satisfied. This completes the proof.

**Theorem 3** clearly shows that Theorem 1 is less conservative than Theorem 2 when using them to analyze the $H_\infty$ performance of the system (17). If the newly proposed bounding technique which provides a tighter estimation for some finite-sum terms appearing in the forward difference of the chosen Lyapunov functional is adopted to deal with the problems in [8,23], less conservative results are expected to be derived. The detailed proof is omitted here for briefness.

Notice that $H_\infty$ performance analysis for an NCS with limited channels is studied in Theorem 1. We now turn to $H_\infty$ performance analysis for an NCS with both limited channels and controller-to-actuator data drift.

**Theorem 4.** For given scalars $\mu, \lambda, l_m, m, \rho, \gamma, d$ and $e$, and the controller gain $K_o$, the system (19) is asymptotically stable in the sense of mean-square with an $H_\infty$ norm bound $\gamma$ if there exist symmetric positive definite matrices $P, Q_1, Q_2, Q_3, Q_4, Q_5, Z_1, Z_2, Z_3$ and $Z_4$, scalars $\varepsilon > 0$, such that the following LMIs hold for every feasible value of $W_{sk}, v_{k1}$ and $v_{k2}$

$$\begin{bmatrix} H & M & \varepsilon N^T \\ \ast & -\varepsilon e I & 0 \\ \ast & \ast & -\varepsilon e I \end{bmatrix} < 0$$

(30)
where

$$
H = \begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} \\
\ast & \Omega_{22} & 0 \\
\ast & \ast & \Omega_{33}
\end{bmatrix}
$$

$$
\Omega_{11}, \Omega_{22}, \text{and } \Omega_{33} \text{ are the same as the corresponding matrices in (25); } \Omega_{12} \text{ is derived from } \Omega_{12} \text{ in (26) by replacing } J_{25} \text{ with } \tilde{y}K_2^T \tilde{W}_{ab} B_1^T; \Omega_{13} \text{ is derived from } \Omega_{13} \text{ in (27) by replacing } J_{25} \text{ and } J_{48} \text{ with } \tilde{y}K_2^T \tilde{W}_{ab} B_1^T \text{ and } \tilde{y}K_2^T \tilde{W}_{ab} D_1^T, \text{ respectively; and}
$$

$$
M_\delta = \begin{bmatrix}
0_{1,7} & 0_{1,5} & 0_{1,5} & 0 & 0 \\
0_{1,7} & \lambda \mathcal{A}_\delta & \mathcal{J}_{25} & \mathcal{J}_{75} \\
0_{1,7} & (1 - \tilde{\lambda}) \mathcal{A}_\delta & -\mathcal{A}_\delta & (1 - \tilde{\lambda}) \mathcal{J}_{75} & -\mathcal{J}_{75} \\
0_{16,7} & 0_{16,5} & 0_{16,5} & 0_{16,1} & 0_{16,1}
\end{bmatrix}
$$

$$
\mathcal{A}_\delta = [J_{65} \; J_{65} \; J_{65} \; J_{65} \; J_{65}], \quad J_{65} = \tilde{e}K_2^T \tilde{W}_{ab} B_1^T \\
J_{75} = \tilde{e}K_3^T \tilde{W}_{ab} D_1^T, \quad N = \text{diag}(0, d, d, 0, \ldots, 0)
$$

**Proof.** For the system (19), considering that \(\tilde{y}, \tilde{d}, \tilde{e}\) are scalars, one can rewrite (25) as

$$
H + M_\delta \tilde{F}_k N + N^T \tilde{F}_k^T M_\delta^T < 0
$$

where \(H, M_\delta\), and \(N\) are the same as the corresponding matrices in (31)-(33), and \(\tilde{F}_k = \text{diag}(f_k, \ldots, f_k)\). From (34), in view of Schur complement, one can see that if the LMIs in (30) are satisfied, the inequalities in (34) are also satisfied. This completes the proof.

4. Controller design

This section is to address controller design for an NCS with both limited channels and controller-to-actuator data drift. We begin with controller design for the NCS with limited channels. Based on Theorem 1, we can derive the following result.

**Theorem 5.** For given scalars \(\mu, \tilde{\lambda}, L_M, L_m, \rho\) and \(\gamma\), the system (17) is asymptotically stable in the sense of mean-square with an \(H_\infty\) norm bound \(\gamma\) and the controller gain \(K_\delta = G_\delta W^{-1}\) if there exist symmetric positive definite matrices \(W, Q_1, Q_2, Q_3, Q_4, Q_5, Z_1, Z_2, Z_3\) and \(Z_4\), and matrices \(G_{ij}\), such that the following LMIs hold for every feasible value of \(W_{ab}, v_{k1}\) and \(v_{k2}\)

$$
\begin{bmatrix}
\tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} \\
\ast & \tilde{\Omega}_{22} & 0 \\
\ast & \ast & \tilde{\Omega}_{33}
\end{bmatrix} < 0
$$

where

$$
\tilde{\Omega}_{11} = \tilde{I} - v_{k1} \frac{\rho - L}{L - L_m} \Phi_1 - (1 - v_{k1}) \frac{L - L_m}{\rho - L} \tilde{\Phi}_2 - v_{k2} \frac{L - \bar{\rho}}{L - \bar{\rho}} \tilde{\Phi}_3 - (1 - v_{k2}) \frac{\bar{\rho} - \rho}{\tilde{L} - \bar{\rho}} \tilde{\Phi}_4
$$

and

$$
\tilde{I} =
\begin{bmatrix}
\tilde{I}_{11} & 0 & 0 & \tilde{Z}_4 & 0 & \tilde{Z}_3 & 0 \\
* & \tilde{I}_{12} & \tilde{Z}_1 & \tilde{Z}_1 & 0 & 0 \\
* & * & \tilde{I}_{13} & 0 & \tilde{Z}_2 & \tilde{Z}_2 & 0 \\
* & * & * & \tilde{I}_{14} & 0 & 0 & 0 \\
* & * & * & * & \tilde{I}_{15} & 0 & 0 \\
* & * & * & * & * & \tilde{I}_{16} & 0 \\
* & * & * & * & * & \ast & -\gamma I
\end{bmatrix}
$$
\[ \begin{align*}
\mathbf{\bar{\Omega}}_{11} &= -\mathbf{W} + (\rho - L_m + 1)\mathbf{\bar{Q}}_1 + (L_m - \rho + 1)\mathbf{\bar{Q}}_2 + \mathbf{\bar{Q}}_3 + \mathbf{\bar{Q}}_4 + \mathbf{\bar{Q}}_5 - \mathbf{\bar{Z}}_1 - \mathbf{\bar{Z}}_2 - \mathbf{\bar{Z}}_3 - \mathbf{\bar{Z}}_4 \\
\mathbf{\bar{\Omega}}_{22} &= -\mathbf{\bar{Q}}_1 - 2\mathbf{\bar{Z}}_1, \quad \mathbf{\bar{\Omega}}_{33} = -\mathbf{\bar{Q}}_2 - 2\mathbf{\bar{Z}}_2, \quad \mathbf{\bar{\Omega}}_{44} = -\mathbf{\bar{Q}}_3 - \mathbf{\bar{Z}}_1 - \mathbf{\bar{Z}}_2 \\
\mathbf{\bar{\Omega}}_{55} &= -\mathbf{\bar{Q}}_4 - \mathbf{\bar{Z}}_1 - \mathbf{\bar{Z}}_2, \quad \mathbf{\bar{\Omega}}_{66} = -\mathbf{\bar{Q}}_5 - \mathbf{\bar{Z}}_2 - \mathbf{\bar{Z}}_3 \\
\mathbf{\bar{\Phi}}_1 &= [0 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0]^{T}\mathbf{\bar{Z}}_1 [0 \quad -1 \quad 0 \quad 0 \quad 0] \\
\mathbf{\bar{\Phi}}_2 &= [0 \quad 1 \quad 0 \quad 0 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^{T}\mathbf{\bar{Z}}_2 [0 \quad 1 \quad 0 \quad 0 \quad -1 \quad 0] \\
\mathbf{\bar{\Phi}}_3 &= [0 \quad 0 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^{T}\mathbf{\bar{Z}}_2 [0 \quad 0 \quad -1 \quad 0 \quad 0 \quad 0] \\
\mathbf{\bar{\Phi}}_4 &= [0 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^{T}\mathbf{\bar{Z}}_2 [0 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0] \\
\end{align*} \]

and

\[ \begin{align*}
\mathbf{\bar{\Omega}}_{12} &= \begin{bmatrix} 
\mathbf{W}^{T} & \mathbf{J}_{1\delta}^{T} & \mathbf{J}_{2\delta}^{T} & \mathbf{J}_{3\delta}^{T} & \mathbf{J}_{4\delta}^{T} \\
\lambda \mathbf{J}_{2\delta}^{T} & \lambda \mathbf{J}_{2\delta}^{T} & \lambda \mathbf{J}_{2\delta}^{T} & \lambda \mathbf{J}_{2\delta}^{T} & \lambda \mathbf{J}_{2\delta}^{T} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\mathbf{B}_{2}^{T} & \mathbf{B}_{2}^{T} & \mathbf{B}_{2}^{T} & \mathbf{B}_{2}^{T} & \mathbf{B}_{2}^{T} 
\end{bmatrix} \\
\mathbf{\bar{\Omega}}_{13} &= \begin{bmatrix} 
0 & 0 & 0 & 0 & 0 & \mathbf{W}^{T} & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{2\delta}^{T} & \mathbf{J}_{2\delta}^{T} \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{2\delta}^{T} & \mathbf{J}_{2\delta}^{T} \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{2\delta}^{T} & \mathbf{J}_{2\delta}^{T} \\
0 & 0 & 0 & 0 & 0 & (1 - \lambda) \mathbf{J}_{2\delta}^{T} & -\mathbf{J}_{2\delta}^{T} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix} \\
\mathbf{\bar{\Omega}}_{22} &= \text{diag}(-\mathbf{W}, \mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4), \quad \lambda = \bar{\lambda} (1 - \bar{\lambda}) \\
\mathbf{\bar{\Omega}}_{33} &= \text{diag}(-\lambda^{-1}\mathbf{W}, \lambda^{-1}\mathbf{X}_1, \lambda^{-1}\mathbf{X}_2, \lambda^{-1}\mathbf{X}_3, \lambda^{-1}\mathbf{X}_4, -\gamma \mathbf{I}, -\bar{\lambda}^{-1} \gamma \mathbf{I}) \\
\mathbf{J}_{1\delta} &= \mathbf{W}(A - I)^{T}, \quad \mathbf{J}_{2\delta} = \mathbf{G}_w \mathbf{W}_{ik}^{T}, \quad \mathbf{J}_{3\delta} = (1 - \bar{\lambda}) \mathbf{J}_{2\delta}, \quad \mathbf{J}_{4\delta} = \mathbf{G}_w \mathbf{W}_{ik}^{T} \mathbf{D}^{T} \\
\mathbf{X}_1 &= (\rho - L_m)^{-2} (\mu \mathbf{Z}_1 - 2 \mu \mathbf{W}), \quad \mathbf{X}_2 = (L_m - \rho)^{-2} (\mu \mathbf{Z}_2 - 2 \mu \mathbf{W}) \\
\mathbf{X}_3 &= L_m^{-2} (\mu \mathbf{Z}_3 - 2 \mu \mathbf{W}), \quad \mathbf{X}_4 = L_m^{-2} (\mu \mathbf{Z}_4 - 2 \mu \mathbf{W}), \quad \mathbf{J}_{3\delta} = -\mathbf{J}_{2\delta} 
\end{align*} \]

**Proof.** In view of Schur complement, one can see that (25) is equivalent to

\[ \begin{bmatrix}
\mathbf{\Omega}_{11} & \mathbf{\Omega}_{12} & \mathbf{\Omega}_{13} \\
* & \mathbf{\Omega}_{22} & 0 \\
* & * & \mathbf{\Omega}_{33}
\end{bmatrix} < 0 \]

where

\[ \mathbf{\Omega}_{12} = \begin{bmatrix} 
\mathbf{A}^{T} & \mathbf{J}_{1\delta} & \mathbf{J}_{1\delta} & \mathbf{J}_{3\delta} & \mathbf{J}_{3\delta} \\
\lambda \mathbf{J}_{2\delta}^{T} & \lambda \mathbf{J}_{2\delta}^{T} & \lambda \mathbf{J}_{2\delta}^{T} & \lambda \mathbf{J}_{2\delta}^{T} & \lambda \mathbf{J}_{2\delta}^{T} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\mathbf{B}_{2}^{T} & \mathbf{B}_{2}^{T} & \mathbf{B}_{2}^{T} & \mathbf{B}_{2}^{T} & \mathbf{B}_{2}^{T} 
\end{bmatrix} \]
and $\Omega_{13} = J_{25} J_{26} J_{28} J_{28} J_{28} \tilde{J}_{45} J_{49}$.

\[
\Omega_{22} = \text{diag}(-P^{-1}, \Psi_4)
\]

\[
\Omega_{33} = \text{diag}(-\tilde{\lambda}^{-1} P^{-1} \Psi_1, \tilde{\lambda}^{-1} \Psi_2, \tilde{\lambda}^{-1} \Psi_3, \tilde{\lambda}^{-1} \Psi_4, -\gamma_1, -\tilde{\lambda}^{-1} \gamma_1)
\]

Algorithm 1.

1. **Step 1.** For given scalars $\tilde{\lambda}, \lambda, L_m, l_m$ and $\rho$, choose the initial value $\mu_0$, the final value $\mu_{\text{ult}}$ ($\mu_{\text{ult}} < \mu_0$) and an appropriate step length $\mu_{\text{dec}} > 0$ for $\mu$. Choose a large enough $H_\infty$ norm bound $\gamma_{\text{opt}}$. Set $\mu_{\text{opt}} = \mu_0$.

2. **Step 2.** Solve the LMIs presented in (35). If $\gamma < \gamma_{\text{opt}}$, set $\gamma_{\text{opt}} = \gamma$; $\mu_{\text{opt}} = \mu$ and go to step 3; otherwise, go to step 3 directly.

3. **Step 3.** Set $\mu = \mu - \mu_{\text{dec}}$. If $\mu \geq \mu_{\text{ult}}$, go to step 2; otherwise, go to step 4.

4. **Step 4.** Output the locally optimal $\mu_{\text{opt}}$ and $\gamma_{\text{opt}}$.

In the case that $\mu = 1$, the inequalities $-Z_j^{-1} \leq \mu^2 P^{-1} Z_j P^{-1} - 2 \mu P^{-1}$ ($j = 1, \ldots, 4$) reduce to $-Z_j^{-1} \leq P^{-1} Z_j P^{-1} - 2 P^{-1}$. One can clearly see that Algorithm 1 can provide better results than choosing $\mu = 1$.

For an NCS with both limited channels and controller-to-actuator data drift, similar to proofs of Theorem 4 and Theorem 5, we have the following channel utilization-based controller design criterion.

**Theorem 6.** For given scalars $\mu, \tilde{\lambda}, \lambda, l_m, \rho, \tilde{\gamma}, \tilde{\delta}$ and $\gamma$, the system (19) is asymptotically stable in the sense of mean-square with an $H_\infty$ norm bound $\gamma$ and the controller gain $K_\beta = G_\beta W^{-1}$ if there exist symmetric positive definite matrices $W, Q_1, Q_2, Q_3, Q_4, Q_5, Z_1, Z_2, Z_3$ and $Z_4$, and matrices $G_\delta$, scalars $\delta > 0$, such that the following LMIs hold for every feasible value of $W_d$, $\psi_{k_1}$ and $\psi_{k_2}$

\[
\begin{bmatrix}
\tilde{H} & \tilde{M}_d \\
-\tilde{\psi}_{k_1} N & 0
\end{bmatrix} < 0
\]

where

\[
\tilde{H} = \begin{bmatrix}
\tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} \\
\ast & \tilde{\Omega}_{22} & 0 \\
\ast & \ast & \tilde{\Omega}_{33}
\end{bmatrix}
\]

and $\tilde{\Omega}_{11}, \tilde{\Omega}_{22}$ and $\tilde{\Omega}_{33}$ are the same as the corresponding matrices in (35); $\tilde{\Omega}_{12}$ is derived from $\Omega_{12}$ in (36) by replacing $J_{26}$ with $\tilde{\psi} G_{\delta} W_d B_1^T$; $\tilde{\Omega}_{13}$ is derived from $\Omega_{13}$ in (37) by replacing $J_{26}$ and $J_{46}$ with $\tilde{\psi} G_{\delta} W_d B_1^T$ and $\tilde{\psi} G_{\delta} W_d D^T$, respectively; and
\[
\tilde{M}_d = \begin{bmatrix}
0_{1,7} & 0_{1,5} & 0_{1,5} & 0 & 0 \\
0_{1,7} & \tilde{\lambda}\tilde{\mathcal{A}}_d & \tilde{\mathcal{A}}_d & \tilde{J}_{75} & \tilde{J}_{75} \\
(1 - \tilde{\lambda})\tilde{\mathcal{A}}_d & -\tilde{\mathcal{A}}_d & (1 - \tilde{\lambda})\tilde{J}_{75} & -\tilde{J}_{75} \\
0_{16,7} & 0_{16,5} & 0_{16,5} & 0_{16,1} & 0_{16,1}
\end{bmatrix}^T
\]  

(43)

\[
\tilde{\mathcal{A}}_d = [\tilde{J}_{65} \tilde{J}_{65} \tilde{J}_{65} \tilde{J}_{65} \tilde{J}_{65}], \quad \tilde{J}_{65} = \tilde{e}G_t\tilde{W}_{ak}\tilde{B}_l^T \\
\tilde{N} = \text{diag}(0, \hat{d}, \hat{d}, 0, \ldots, 0)
\]  

(44)

It should be pointed out that the proposed controller design in Theorem 6 can enhance robustness of the NCS (19) to data drift.

5. Simulation results and discussion

In this section, we give an example to show the effectiveness of the newly proposed channel utilization-based controller design. Consider the following open-loop unstable NCS

\[
\begin{bmatrix}
0.7566 & -0.0591 \\
0.6149 & 1.1704
\end{bmatrix} x_k + \begin{bmatrix}
0.1126 & 0.4238 \\
-0.2959 & 0.9067
\end{bmatrix} u_k + \begin{bmatrix}
-0.6522 \\
0.1915
\end{bmatrix} \omega_k
\]

(45)

\[
z_k = [-1.1074 -0.6290] x_k + [-0.3473 -1.1826] u_k
\]

Suppose that \( L_M = 5, L_m = 1, \tilde{\lambda} = 0.9, \tilde{y} = 1, \hat{d} = 0.5, \tilde{\epsilon} = -0.5 \), and the system can provide simultaneously one controller-to-actuator communication channel, which means that \( \tilde{W}_{ak} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \) or \( \tilde{W}_{ak} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \). From \( L_M = 5, L_m = 1 \), one has \( \rho = 3, \tilde{I} = 2, \hat{\rho} = 4 \).

For the system (45), by applying Theorem 5, we obtain the optimal \( H_\infty \) norm bound \( \gamma \) for different \( \mu \) and list the results in Table 1.

As shown in Table 1, if an appropriate \( \mu \) is chosen, less conservative results can be obtained when compared with choosing \( \mu = 1 \). Suppose that \( \mu_0 = 3, \mu_{\text{act}} = 2, \mu_{\text{dec}} = 0.1 \). By using Algorithm 1, one can obtain the locally optimal \( \mu_{\text{opt}} = 3 \) with the \( H_\infty \) norm bound \( \gamma_{\text{opt}} = 25.1331 \), which illustrates the effectiveness of Algorithm 1.

The open-loop eigenvalues of the system (45) are 0.8831 and 1.0439. As analyzed in Remark 7, if more transmission priority is allocated to the 2nd element of \( u_k \), better system performance can be achieved. Suppose that the communication channels scheduling scheme given in Remark 7 and the locally optimal \( \mu_{\text{opt}} = 3 \) are adopted. Then the \( H_\infty \) norm bounds corresponding to \( \tilde{W}_{ak} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \) and \( \tilde{W}_{ak} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \) are \( \gamma_1 = 25.1331 \) and \( \gamma_2 = 9.8464 \), respectively. From \( \gamma_2 < \gamma_1 \), one can conclude that the channels scheduling scheme given in Remark 7 is effective.

For the locally optimal \( \mu_{\text{opt}} = 3 \), solving the LMIs in (35), one can see that the controller gains corresponding to \( \tilde{W}_{ak} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \) and \( \tilde{W}_{ak} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \) are \( \tilde{K}_1 = \begin{bmatrix} 4.1302 & 1.9413 \\ 0 & 0 \end{bmatrix} \) and \( \tilde{K}_2 = \begin{bmatrix} -0.1471 & -0.0694 \\ 0 & 0 \end{bmatrix} \), respectively. Suppose that the initial state of the system (45) is \( x_0 = [0.1 \ 0.1]^T \). The lump sum of packet dropouts \( p_k \) and network-induced delays \( d_k \), that is \( \tilde{d}_k \), is shown in Fig. 1. The switching law of controller gains is shown in Fig. 2 \((\sigma_k = 1 \ and \ \sigma_k = 2 \ denote \ that \ the \ controller \ gains \ K_1 \ and \ K_2 \ are \ adopted, \ respectively)\). The disturbance \( \omega_k \) is defined by

\[
\omega_k = \begin{cases}
-0.1, & 0 \leq k \leq 7 \\
0, & \text{otherwise}
\end{cases}
\]  

(46)

Then the plant state response and controlled output are given in Fig. 3. From Fig. 3, one can see that even if the available communication channels are limited, the newly proposed controller design is still effective.

In the case that considering both limited channels and controller-to-actuator data drift, by solving the LMIs in (41) with \( \mu = 3 \), one can see that the controller gains corresponding to \( \tilde{W}_{ak} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \) and \( \tilde{W}_{ak} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \) are \( \tilde{K}_1 = \begin{bmatrix} 4.0699 & 1.9176 \\ 0 & 0 \end{bmatrix} \) and \( \tilde{K}_2 = \begin{bmatrix} -0.1427 & -0.0672 \\ 0 & 0 \end{bmatrix} \), respectively. Suppose that the lump sum of packet dropouts and network-induced delays,

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>2</th>
<th>2.2</th>
<th>2.6</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>27.1000</td>
<td>26.4035</td>
<td>25.5348</td>
<td>25.1331</td>
</tr>
</tbody>
</table>

Table 1: \( H_\infty \) norm bound \( \gamma \) for different \( \mu \).
Fig. 1. The lump sum of $p_k$ and $d_k$.

Fig. 2. The switching law of controller gains.

Fig. 3. The plant state response and controlled output.
and the switching law of controller gains are the same as the ones given in Figs. 1 and 2, respectively. The disturbance \( d_k \) is given in (46). The data drift items \( \delta_k = \tilde{y} + \tilde{d}_k \tilde{e} \), where \( f(k) \) is defined by

\[
f(k) = \begin{cases} 
\cos(k + 1), & 1 \leq k \leq 7 \\
0, & \text{otherwise}
\end{cases}
\]

(47)

Then the plant state response and controlled output are plotted in Fig. 4. As illustrated in Fig. 4, the newly proposed controller design in Theorem 6 can enhance robustness of the considered NCS to data drift and external disturbances.

As analyzed in Section 3, if the newly proposed bounding technique which provides a tighter estimation for some finite-sum terms appearing in the forward difference of the chosen Lyapunov functional is adopted to deal with the problems in [8,23], better results are expected to be derived. Since the system considered in this paper is different from the systems in [8,23], the numerical comparison between the results in this paper and the results in [8,23] is omitted.

6. Conclusions

The problem of modelling, \( H_\infty \) performance analysis and controller design for a discrete-time NCS with both limited channels and controller-to-actuator data drift has been investigated. The main contributions of this paper are summarized as follows: a new model for a discrete-time NCS with both limited channels and controller-to-actuator data drift has been established; the channel utilization-based switched controllers for the considered NCS have been introduced; and a controller design criterion, which can enhance robustness of an NCS to data drift and external disturbances, has been derived. When dealing with the finite-sum terms appearing in the forward difference of the chosen Lyapunov functional, a new bounding technique has been proposed, and the theoretical proof has illustrated that the newly proposed bounding technique can provide a tighter estimation. Notice that the controller-to-actuator data drift has been considered in this paper. Even for an NCS considering both sensor-to-controller and controller-to-actuator data drift, the proposed modelling and controller design in this paper are still applicable.

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Appendix A. Proof of Lemma 1

Notice that

\[
-(R_M - R_r) \sum_{i=k-T_M}^{k-T_M+1} W_i Z_i - (R_M - R_m) \sum_{i=k-T_M}^{k-T_M+1} W_i Z_i = -(R_M - R_r) \sum_{i=k-T_M}^{k-T_M+1} W_i Z_i
\]

\[
-(R_M - R_r) \sum_{i=k-T_M}^{k-T_M+1} W_i Z_i - (R_M - R_m) \sum_{i=k-T_M}^{k-T_M+1} W_i Z_i = -(R_M - R_r) \sum_{i=k-T_M}^{k-T_M+1} W_i Z_i - (R_M - R_r) \sum_{i=k-T_M}^{k-T_M+1} W_i Z_i
\]
By using Lemma 1 in [8], one has

\[
-(r_k - r_m) \sum_{i=k-r_k}^{k-r_m-1} \eta_i^T Z \eta_i \leq \lambda \|Z\|_1 \|Y_1\|_k
\]

(A.1)

and

\[
-(r_M - r_k) \sum_{i=k-r_M}^{k-r_k-1} \eta_i^T Z \eta_i = -(r_M - r_k) \sum_{i=k-r_M}^{k-r_k-1} \eta_i^T Z \eta_i \leq -(r_M - r_k) \lambda \|Z\|_1 \|Y_1\|_k
\]

(A.2)

Similarly, one has

\[
-(r_m - r_k) \sum_{i=k-r_m}^{k-r_k-1} \eta_i^T Z \eta_i \leq -(r_m - r_k) \lambda \|Z\|_1 \|Y_1\|_k
\]

(A.3)

Combining the inequalities (A.1)–(A.4) together, one can derive (23). This completes the proof.

**Appendix B. Proof of Theorem 1**

Taking the time difference of the Lyapunov functional \(V(x_k, k)\) in (22) along the trajectory of the system (17), and considering that \(\Delta V(x_k, k) = \sum_{i=1}^{g} \Delta V_i(x_k, k) = \sum_{i=1}^{g} E(V_i(x_{k-1}, k + 1)\{x_k\}) - V(x_k, k), E(\dot{x}_k - \dot{x}_k) = 0, E(\dot{x}_k - \dot{x}_k)^2 = \bar{\lambda}(1 - \bar{\lambda}), \) one has

\[
\Delta V_1(x_k, k) = \phi_1^T P \phi_1 + 2 \phi_1^T P B_2 \phi_1 + E(\dot{x}_k - \dot{x}_k)^2 \phi_1^T P \phi_1 + \alpha_k^2 \phi_1^T P \phi_1 - \phi_1^T \rho - \frac{\rho^T \phi_1}{\rho} \phi_1^T Z \phi_1
\]

(B.1)

\[
\Delta V_2(x_k, k) \leq (\rho - L_m + 1) q \phi_2^T P \phi_2 + E(\dot{x}_k - \dot{x}_k)^2 \phi_2^T P \phi_2 + \alpha_k^2 \phi_2^T P \phi_2 - \phi_2^T \rho - \frac{\rho^T \phi_2}{\rho} \phi_2^T Z \phi_2
\]

(B.2)

\[
\Delta V_3(x_k, k) \leq (\rho - L_\rho + 1) q \phi_{3k}^T P \phi_{3k} + E(\dot{x}_k - \dot{x}_k)^2 \phi_{3k}^T P \phi_{3k} + \alpha_k^2 \phi_{3k}^T P \phi_{3k} - \phi_{3k}^T \rho - \frac{\rho^T \phi_{3k}}{\rho} \phi_{3k}^T Z \phi_{3k}
\]

(B.3)

\[
\Delta V_4(x_k, k) = \phi_1^T (Q + \phi_1) x_k - \phi_2^T (Q + \phi_2) x_k - \phi_3^T (Q + \phi_3) x_k - x_k^T \rho - \phi_1^T \phi_1 - \phi_2^T \phi_2 - \phi_3^T \phi_3
\]

(B.4)

By using Lemma 1, one has

\[
\Delta V_5(x_k, k) = (\rho - L_m)^2 E((x_{k+1} - x_k)^T Z (x_{k+1} - x_k)) - (\rho - L_m) \sum_{i=k-L_1}^{k-L_{m-1}} \eta_i^T Z \eta_i - (\rho - L_m) \sum_{i=k-L}^{k-L_{m-1}} \eta_i^T Z \eta_i
\]

\[
\leq (\rho - L_m)^2 \left[ \phi_1^T Z \phi_1 + 2 \phi_1^T Z \phi_2 + E(\dot{x}_k - \dot{x}_k)^2 \phi_1^T Z \phi_1 + \alpha_k^2 \phi_1^T Z \phi_1 - \phi_1^T \rho - \frac{\rho^T \phi_1}{\rho} \phi_1^T Z \phi_1 \right]
\]

(B.5)

where \(\phi_{1k} = (A - I)x_k + ZB_1 \bar{W}_{1k} K x_{k-L_1} + (1 - \bar{\lambda}) B_1 \bar{W}_{1k} K x_{k-L_2}, \quad \phi_{2k} = x_k - \phi_{1k}, \quad \phi_{3k} = x_k - \phi_{1k}, \quad \phi_{4k} = x_k - \phi_{1k}, \quad \phi_{5k} = x_k - \phi_{1k}, \) Similar to \(\Delta V_5(x_k, k),\) one has

\[
\Delta V_6(x_k, k) = (L_m - \rho)^2 E((x_{k+1} - x_k)^T Z (x_{k+1} - x_k)) - (L_m - \rho) \sum_{i=k-L_1}^{k-L_{m-1}} \eta_i^T Z \eta_i - (L_m - \rho) \sum_{i=k-L}^{k-L_{m-1}} \eta_i^T Z \eta_i
\]

\[
\leq (L_m - \rho)^2 \left[ \phi_{1k}^T Z \phi_{1k} + 2 \phi_{1k}^T Z \phi_{2k} + E(\dot{x}_k - \dot{x}_k)^2 \phi_{1k}^T Z \phi_{2k} + \alpha_k^2 \phi_{1k}^T Z \phi_{2k} - \phi_{1k}^T \rho - \frac{\rho^T \phi_{1k}}{\rho} \phi_{1k}^T Z \phi_{1k} \right]
\]

(B.6)

where \(\phi_{5k} = (x_k - \phi_{1k}), \quad \phi_{6k} = (x_k - \phi_{1k}), \) \(\phi_{7k} = (x_k - \phi_{1k}), \quad \phi_{8k} = (x_k - \phi_{1k}), \) \(\phi_{9k} = (x_k - \phi_{1k}), \) \(\phi_{10k} = (x_k - \phi_{1k}),\)

\[
\Delta V_7(x_k, k) = L_m^2 E((x_{k+1} - x_k)^T Z (x_{k+1} - x_k)) - L_m \sum_{i=k-L_1}^{k-L_{m-1}} \eta_i^T Z \eta_i
\]

\[
\leq L_m^2 \left[ \phi_{1k}^T Z \phi_{1k} + 2 \phi_{1k}^T Z \phi_{2k} + E(\dot{x}_k - \dot{x}_k)^2 \phi_{1k}^T Z \phi_{2k} + \alpha_k^2 \phi_{1k}^T Z \phi_{2k} - (x_k - \phi_{1k})^T Z (x_k - \phi_{1k}) \right]
\]

(B.7)
\[\Delta V_h(x_k, k) = \sum_{i=k}^{k-1} \eta_i^T Z_4 \eta_i \leq L_m^2 \langle \dot{z}_k \rangle Z_4 \dot{z}_k + 2 \delta \dot{z}_k Z_4 \dot{z}_k + E\{\dot{z}_k \dot{z}_k^T\} + \omega_k^T B_2^T Z_4 B_2 \omega_k - (x_k - x_{k-l_m})^T Z_4 (x_k - x_{k-l_m}) \]  

(B.8)

Then, by combining (17), (B.1)–(B.8) together, and considering that \( E\{\dot{z}_k \dot{z}_k^T\} = \dot{\lambda}(1 - \dot{\lambda}) \), one has

\[ E\{\Delta V(x_k, k)\} + E\{\gamma \langle \dot{z}_k \dot{z}_k^T\rangle - \gamma \omega_k^T \bar{w}_k \leq E\{\gamma \Omega \dot{z}_k \} \]  

(B.9)

where \( \bar{z}_k = \begin{bmatrix} x_k^T & x_{k-l_m}^T & x_{k-l_m}^T & x_{k-l_m}^T & \omega_k^T \end{bmatrix}^T \), \( \Omega = \Pi + \Xi_1 + \Xi_2 \), \( \Pi \) and \( \Xi_2 \) are the same as the corresponding matrices in (25), and

\[ \Xi_1 = \gamma T_1 \Pi T_1 + \gamma T_2 \Pi T_2 + \lambda(1 - \dot{\lambda}) T_3 \]  

\[ T_1 = \begin{bmatrix} A & J_{23} & 0 & 0 & 0 & B_2 \\ \lambda J_{23} & (1 - \dot{\lambda}) J_{23} & 0 & 0 & 0 & B_2 \\ 0 & J_{23} & 0 & 0 & 0 & 0 \\ C & \lambda D W_{ik} K_3 & 0 & 0 & 0 & 0 \\ 0 & D W_{ik} K_3 & 0 & 0 & 0 & 0 \\ A &= \begin{bmatrix} \rho - L_m^2 Z_1 + (L_m - \rho) \lambda Z_2 + L_m^2 \lambda Z_2 + L_m^2 \lambda Z_2 \end{bmatrix} \]

with \( J_{23} = K_{21} \bar{W}_k \bar{W}_k^T \).

From \( \Omega = \Pi + \Xi_1 + \Xi_2 \), one can see that if \( \Pi + \Xi_1 + \Xi_2 < 0 \), then \( E\{\Delta V(x_k, k)\} + E\{\gamma \langle \dot{z}_k \dot{z}_k^T\rangle - \gamma \omega_k^T \bar{w}_k \leq E\{\gamma \Omega \dot{z}_k \} < 0 \). In view of Schur complement, one can see that \( \Pi + \Xi_1 + \Xi_2 < 0 \) is equivalent to (25). Then if the LMIs in (25) are satisfied, one has

\[ E\{\Delta V(x_k, k)\} + E\{\gamma \langle \dot{z}_k \dot{z}_k^T\rangle - \gamma \omega_k^T \bar{w}_k < 0 \]  

(B.10)

Summing up (B.10) from 0 to \( \infty \) with respect to \( k \), one has

\[ \sum_{k=0}^{\infty} E\{||z_k||^2\} < \gamma \sum_{k=0}^{\infty} E\{||\omega_k||^2\} + \gamma E\{V(x_0, 0) - \gamma E\{V(x_\infty, \infty)\} \} \]  

(B.11)

If the disturbance \( \omega_k \), (25) can guarantee the asymptotic stability of the system (17) in the sense of mean-square. Under the zero initial condition, if \( \omega_k \neq 0 \), one has \( \sum_{k=0}^{\infty} E\{||z_k||^2\} < \gamma^2 \sum_{k=0}^{\infty} E\{||\omega_k||^2\} \).

Then if the LMIs in (25) are satisfied, the system (17) is asymptotically stable in the sense of mean-square with an \( H_\infty \) norm bound \( \gamma \). This completes the proof.

References


